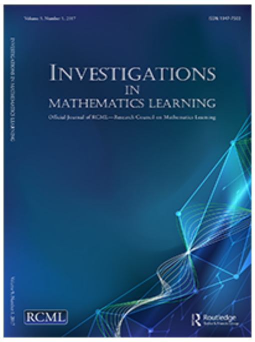

Investigations in Mathematics Learning

ISSN: 1947-7503 (Print) 2472-7466 (Online) Journal homepage: https://www.tandfonline.com/loi/uiml20

The Influence of Students' Self-Perceptions and Mathematics Experiences on Learning More Mathematics in the Future

Jeffrey Shih, Marsha Ing, Julia Phelan, Richard Brown & Cathrine Maiorca

INVESTIGATIONS IN MATHEMATICS LEARNING
https://doi.org/10.1080/19477503.2019.1582960

The Influence of Students' Self-Perceptions and Mathematics Experiences on Learning More Mathematics in the Future

Jeffrey Shih [a], Marsha Ing [b], Julia Phelan[c], Richard Brown[d], and Cathrine Maiorca[e]

[a]College of Education University of Nevada, Las Vegas, NV; [b]Graduate School of Education University of California, Riverside; [c]National Center for Research on Evaluation, Standards, and Student Testing; [d]West Coast Analytics, Irvine, CA; [e]California State University, Long Beach

ABSTRACT
This study explores the relationships among students' perceived learning opportunities in school mathematics, their perceptions of self in mathematics, and their desire to learn more mathematics in the future. Data included student survey responses from eighth and tenth grade students from a state in the Southeast region of the United States. Students were surveyed in the fall (n = 299) and spring (n = 310). The same analyses were replicated at each time point. Results were consistent at each time point and showed that students' desire to learn more mathematics in the future was a function not only of the extent to which they had classroom opportunities, but also whether they developed positive perceptions of themselves in mathematics. This has notable implications for teachers because teachers influence both school mathematics activities and students' sense of self as a mathematics learner.

KEYWORDS
Mathematics; self-perceptions; opportunity to learn

Introduction

The significance of mathematics education in middle and high school as it relates to college and career readiness and success is well documented (National Council of Teachers of Mathematics, 2007; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; National Research Council, 2001). For example, algebra is often referred to as the "gateway" to higher education (Gojak, 2013) in that success in algebra provides students with fundamental math concepts upon which future mathematics courses build (Harel, Selden, & Selden, 2006). Mathematics in the middle and high school grades also provides opportunity for students to begin to develop abstract thinking (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005). Unfortunately, not all mathematics teaching leads to student success and persistence in the subject. Teaching and practicing procedures and algorithms comprises the bulk of secondary math education (Donovan, Bransford, & Pellegrino, 1999), which lead to educational experiences that do not promote deep understanding of those algorithms or elaborate on the connections to the non-academic world. As a consequence of these instructional opportunities, many K-12 mathematics students do not construct meaning around core concepts and principles; cannot relate concepts to problem-solving skills and procedures; and view mathematics as a collection of isolated, meaningless procedures to be memorized, not understood (e.g., Carpenter & Lehrer, 1999).

The existence of these beliefs – that math is not interesting or that success in mathematics is unattainable – has led to research focused on better understanding issues of student interest and persistence (see for example, Bobis, Anderson, Martin, & Way, 2011; Eccles & Wang, 2015; Singh, Granville, & Dika, 2002; Wilburne & Dause, 2017). The purpose of the present study was to explore how one particular perspective, students' mathematics self-perceptions, may influence desire to learn more mathematics in the future. This study was informed by previous work with high school students, which highlighted the importance of students' mathematics self-perceptions and their

CONTACT Jeffrey Shih jshih@unlv.nevada.edu

relationship to student classroom experiences and emerging mathematics aspirations (Gaspard et al., 2015). This literature linked self-perceptions to choices of high school math courses (Simpkins, Davis-Kean, & Eccles, 2006), college major (Musu-Gillette, Wigfield, Harring, & Jacquelynne, 2015), and career aspirations (Nagy, Trautwein, Baumert, Köller, & Garret, 2006). This literature also associates self-perceptions with more active processing of information where students both influence and are influenced by their experiences. While previous literature is consistent in the relationship with self-perceptions and mathematics-related outcomes, research on the role of instructional opportunities in influencing both self-perceptions and mathematics-related outcomes is lacking. The role of instructional opportunities is important because it is a malleable factor influenced by the experiences students have in school through interactions with teachers and other students. These opportunities have the potential to not only influence self-perceptions in mathematics, but also to influence long-term mathematics-related outcomes. It is operationalized in a similar way to other large-scale studies (Desmione, Smith, & Frisvold, 2010). This study explores both instructional opportunities and self-perceptions in mathematics for eighth and tenth grade students. More specifically, this study addresses the following research questions:

(1) Do students' mathematics self-perceptions relate to wanting to learn more mathematics?
(2) Do mathematics experiences influence students' mathematics self-perceptions?
(3) Is the influence of mathematics experiences on their wanting to learn more mathematics similar across students with different mathematics self-perceptions?

This study directly addresses concerns about the reproducibility of results of small educational studies, like these, by exploring whether results from one sample is replicated in a second sample (Open Science Collaboration, 2015). To build greater confidence in educational research methodology and findings, this study provides a replication and offers increased openness and transparency in the research process (Collaboration, 2012; Nosek et al., 2015; Open Science Collaboration, 2015).

Students' Mathematics Self-Perceptions

Self-perceptions include "students' thoughts, beliefs, and feelings about themselves, other persons, and events" (Schunk, 1992, p. 4). Self-perceptions include diverse theoretical perspectives. For example, self-perceptions include self-concept, "one's collective self-perceptions that are formed through experiences with, and interpretations of, the environment, and that are heavily influenced by reinforcements and evaluations by significant other persons" (Schunk & Meece, 1992, p. 5) and self-efficacy, an individual's belief of what they think they can do (Bandura, 1977). Self-perceptions also include cognitive perspectives on motivation such as the amount of perceived control that one has over outcomes (Rotter, 1966; Skinner, Wellborn, & Connell, 1990; Wilkie & Sullivan, 2018) or the connection between perceived achievement goals, reward structures, attributions and achievement behaviors (Weiner, 1990). What these different theoretical perspectives have in common is that perceptions represent "complex processes that are influenced by a variety of factors and that have diverse effects in school" (Schunk, 1992, p. 5).

Gaspard et al. (2015) hypothesized that it was possible to change high school students' value beliefs using classroom interventions. These interventions were used to help participants see the relevance of studying mathematics. For one of the interventions, participants were asked to write about the usefulness of mathematics; for the other intervention, students were asked to make connections to their everyday lives by reflecting on arguments written by college students about the utility of mathematics. Unlike previous research, the participants' value beliefs about mathematics did not decrease with time. Musu-Gillette et al. (2015) conducted a study to describe the change in student interest, and beliefs about the value of mathematics and classified their results into three classes: High-Self Concept Trajectory, Slow Decline Self-Concept Trajectory and Fast Decline Self-Concept Trajectory. The researchers found that students with more positive views on their mathematical abilities and values were more likely to select a college major that was mathematics intensive while students in the groups where their self-concepts declined were very unlikely to enroll in majors that required a high usage of mathematics. Researchers did not find significant differences based on gender and achievement for the different classes and that students' interest in mathematics decreased over time. Students with the least interest in mathematics remained uninterested in mathematics over time. However, students in the other two groups became more interested in mathematics in the upper secondary years. There were no significant gender differences between the three groups.

This study considers an expectancy-value achievement motivation approach to understanding self-perceptions. Expectancy-value theory focuses on how students' choices, persistence, and performance are influenced by their beliefs about how well they will do on the activity and the extent to which they value the activity (Atkinson, 1957; Eccles et al., 1983; Wigfield, 1994; Wigfield & Eccles, 1992, 2000). The development of a student's beliefs and values are important with this particular theoretical framework because it influences academic outcomes and choices.

Expectancies are beliefs or expectations about how well one will do on a given task. Pintrich and Schrauben (1992) describe the expectancy component as the answer to the basic question: Can I do this task? (p. 152) and includes "an individuals' beliefs about their ability to perform a task, their judgements of self-efficacy and control, and their expectancy for success at the task" (p. 154). In other words, beliefs about current competencies might answer the question: Can I do this task now?; whereas expectancies for success in the future might answer the question: Do I expect to do this task later this year? (Eccles et al., 1983).

Values include attainment value or importance, intrinsic value, utility value or usefulness of the task and cost (Eccles et al., 1983; Wigfield & Eccles, 1992, 2000). Attainment value is the value or importance one assigns to doing well on a particular task. Intrinsic value is the enjoyment one assigns to doing well on a particular task. Utility value or usefulness is the value one perceives the task fitting future plans and when considering cost, one weighs decisions of how engaging in one task might limit access to another task.

This framework was initially proposed and tested in the mathematics achievement domain (Eccles et al., 1983), but has been widely used in other domains such as science (Eccles & Wigfield, 2002; Schunk, Meece, & Pintrich, 2014; Simpkins et al., 2006). There are consistent relationships between achievement motivation and achievement related choices. Students with more positive expectancy-value achievement motivation are defined as students who believe that they can do a particular task, and are interested in and value the task. Students with lower expectancy-value achievement motivation are defined as students who do not believe they can do a particular task, are not interested in, and do not value the task. In mathematics education, a student with higher achievement motivation is described as one who tends to be more actively engaged in whole class mathematical discussions by volunteering their own thinking or adding on to or challenging the ideas of other students; whereas a lower achievement motivation student might be a student who does not pay much attention to such discussions.

This perspective was selected due to an interest in the role school mathematics experiences play in both students' mathematics self-perceptions and their interest in academic choices (wanting to take more mathematics in the future). In line with the ideas about effective teaching and learning found in NCTM's *Principles to Actions* (National Council of Teachers of Mathematics, 2016) and NRC's *Adding it Up: Helping Children Learn Mathematics* (National Research Council, 2001), this perspective is concerned not only in the tasks in which students engage, but the instructional opportunities and experiences which may foster or hinder their engagement. This requires understanding the opportunities afforded to students in their mathematics classrooms, whether these opportunities promote reasoning and problem solving (National Council of Teachers of Mathematics, 2016) and how these opportunities influence students' self-perceptions and interests in math. This study contributes to the extensive literature on achievement motivation by bringing together self-perceptions of opportunities to learn school mathematics and students' mathematics self-perceptions.

Method

Data included student survey responses from eighth and tenth graders from a state in the Southeast region of the United States. Teachers who volunteered to participate in a statewide professional development program recruited their students to participate in this study. Teachers taught a range of mathematics courses, but only students enrolled in their Algebra I and Algebra II classes were recruited to participate. The reason for focusing on this particular group of students is that student achievement scores were available for these particular grade levels and content areas. Achievement data were analyzed as part of the larger evaluation effort about the influence of professional development on student outcomes. While the teachers were new to this particular professional development program, they were not novice teachers (average number of years teaching was 13.5). There were 67 mathematics teachers from 39 schools in the initial cohort. 73% of the teachers signed letters of agreement to participate in the study. Email reminders were sent by the research project director and professional development coordinators. In addition, the professional development coordinators visited schools and encouraged teachers to participate in the study. However, only 29 teachers recruited students to complete surveys.

A set of forty paper copies of student surveys were mailed to each teacher in the fall and the spring. Teachers were asked to mail the completed surveys back to a central location within three months. After data was entered into electronic format from the paper copies of the student surveys, 20% of the data were randomly checked against the original copies to ensure accuracy. There were 1,111 survey responses received in the fall and 360 survey responses received in the Spring. Approximately 12% of the survey responses were dropped due to missing responses and 8% dropped due to odd responses. An example of an odd response was if a student filled out the same response options for all survey items. In addition, only data from teachers who participated in the Fall and Spring survey administration were included in the analyses which resulted in a final sample size of 299 in the fall and 310 in the spring. Due to the Human Subjects agreement, identifiable information such as the names or student identification numbers was not collected. Without this information, matching student response in the fall and spring was not possible. In addition, matching student responses to other sources of information such as student achievement or student demographic information was also not possible. Student self-reported information on gender and ethnicity was part of the survey but this information could not be verified with other sources. In addition, we were not able to identify students based only on this self-reported information.

Variables Included in Analyses

Students' Perceived Opportunities to Participate in Mathematics

Nine survey items measured students' perceived opportunities to regularly participate in school mathematics activities (Table 1). The items were part of the Organisation for Economic Co-Operation and Development Program for International Student Assessment (PISA) 2012 student questionnaire. PISA is an international survey administered to 15-year old students worldwide. PISA includes information about student performance in reading, mathematics and science, student background, student approaches to learning, and the organization of the schools. The student questionnaire includes questions about the student, their family and home, learning mathematics, their mathematics experiences, and their school. Nine items focused on student mathematics experiences to describe student perceptions of what their mathematics teacher taught were selected (e.g., Thinking about the mathematics teacher that taught your last mathematics class, how often does each of the following happen?). There were four response options: never, in some lessons, in most lessons, in all lessons. The internal consistency for the nine items was lower in the fall (α = .71) compared to the spring (α = .83). A composite variable that combined these items into a single variable referred to as opportunities to learn mathematics was created. A higher value on this variable indicated a student reporting more opportunities to participate in mathematics.

Table 1. Descriptive statistics.

	Fall (n = 299)		Spring (n = 310)	
	Mean	SD	Mean	SD
Perceived Opportunities to Participate in School Mathematics[a]				
My teacher asks questions that make me reflect on the problem.	0.70	0.46	0.74	0.44
My teacher gives problems that require me to think for an extended time.	0.72	0.45	0.75	0.43
My teacher asks me to decide on my own procedures for solving complex problems.	0.45	0.50	0.54	0.50
My teacher presents problems for which there is no immediately obvious method of solution.	0.52	0.51	0.58	0.50
My teacher presents problems in different contexts so that students know whether they have understood the concepts.	0.79	0.41	0.77	0.43
My teacher helps me to learn from my mistakes.	0.82	0.39	0.81	0.40
My teacher asks me to explain how I solved a problem.	0.75	0.43	0.75	0.43
My teacher presents problems that require me to apply what I have learned to new contexts.	0.88	0.33	0.84	0.37
My teacher gives problems that can be solved in different ways.	0.81	0.39	0.83	0.38
Mathematics Self-Perceptions[b]				
I am good at mathematics.	0.85	0.36	0.79	0.41
Compared to other students in my class, I am one of the best in mathematics.	0.48	0.50	0.48	0.50
Compared to my other school subjects, I am better at mathematics.	0.47	0.50	0.44	0.50
I expect to do well in mathematics this year	0.93	0.26	0.88	0.32
I am good at learning new things in mathematics.	0.76	0.43	0.75	0.43
What I learn in mathematics is useful outside of mathematics class.	0.54	0.50	0.55	0.50
For me, being good at mathematics is very important.	0.85	0.36	0.85	0.36
Working on mathematics assignments is very interesting.	0.42	0.49	0.37	0.48
I really like doing mathematics.	0.50	0.50	0.49	0.50

[a]Dichotomous variables where 1 = most lessons, 0 = not most lessons
[b]Dichotomous variables where 1 = agree, 0 = disagree

Students' Mathematics Self-Perceptions

Nine survey items comprised our measure of students' mathematics self-perceptions (Table 1). These items are from the expectancy-value achievement motivation theory framework (Eccles et al., 1983; Wigfield, 1994; Wigfield & Eccles, 1992, 2000), which suggests that student performance and persistence are influenced by their beliefs in their abilities and the extent to which they value the activities that they participate in (Simpkins et al., 2006). There were three items related to ability beliefs (I am good at mathematics; Compared to other students in my class, I am one of the best in mathematics; Compared to my other school subjects, I am better at mathematics), two items related to expectancy beliefs (I expect to do well in mathematics this year; I am good at learning new things in mathematics), and four items related to usefulness, importance, and interest (What I learn in mathematics is useful outside of mathematics class; For me, being good at mathematics is very important; Working on mathematics assignments is very interesting; I really like doing mathematics). Internal consistency of these items were similar in the fall (α = .87) and spring (α = .88). There were four response options for these items: strongly disagree, disagree, agree, and strongly agree. Due to skewed distributions, the response categories were converted into two categories with a "1" indicating that the student "strongly agreed" or "0" indicating that the student did not strongly agree. All nine items were then summed together with higher values on the composite indicated more positive mathematics self-perceptions.

Desire to Study Mathematics in the Future

Students were asked about their desire to study and engage in mathematics in the future (I would like a career involving mathematics; I would like to take more mathematics courses; and I would like to study mathematics after high school. Internal consistency for these three items were similar in the fall (α = .78) and spring (α = .77). The response options for these items were agree (1) or disagree (0). Three items were summed together with higher values on the composite indicated a greater desire to study mathematics in the future.

These variables were used in a series of regression analyses that considered the nested structure of the data. For example, to address the first research question, the relationship between students' mathematics self-perceptions and wanting to learn more mathematics was considered. By estimating the relationship between these two variables after controlling for gender and ethnicity, this provided evidence of whether students' mathematics self-perceptions related to wanting to learn more mathematics. This analyses was replicated for data at both time points.

Initial descriptive information about the data indicates no gender differences in the reporting of opportunities to learn mathematics in the fall ($t(297) = .31$, $p = .75$) or spring ($t(308) = .75$, $p = .45$). There were also no ethnicity differences in terms of student perceived opportunities to learn in the fall ($t(297) = -0.50$, $p = .61$) or spring ($t(308) = -0.06$, $p = .95$). There were also no statistically significant differences by gender or ethnicity for student self-perceptions. There were, however, gender differences for professed interest in mathematics; in the fall, males ($M = 1.78$, $SD = 1.27$) reported more interest in mathematics than females ($M = 1.29$, $SD = 1.20$, $t(297) = 3.42$, $p < .001$). Similarly, in the spring, males ($M = 1.68$, $SD = 1.27$) reported more interest in mathematics than females ($M = 1.27$, $SD = 1.17$, $t(308) = 2.91$, $p < .01$). There were no statistically significant differences by gender for wanting to learn more mathematics in the future. While information about classrooms and teachers were not included in our analyses, the non-independent nature of the student survey data was accounted for by clustering standard errors for students within the same classroom.

Results

There were positive relationships between mathematics self-perceptions and interest in mathematics (Table 2). This relationship was consistent across both samples even after adjusting for gender and ethnicity and for the nested structure of the data. Students with higher mathematics self-perceptions were more likely to want to take more mathematics courses in high school and college and have a career that involves mathematics compared to students with lower mathematics self-perceptions.

There were positive relationships between opportunities to participate in mathematics and mathematics self-perceptions (Table 2) after adjusting for gender and ethnicity and for the nested structure of the data. This positive relationship suggests that students with more opportunities to participate in mathematics have a higher mathematics self-perception. When both perceived learning opportunities and self-perceptions are included in the model, there were significant relationships between self-perceptions and wanting to take more math as well as between perceived learning opportunities and self-perceptions. However, for both time points, the relationship between

Table 2. Summary of regression analysis.

	Relationship Between Mathematics Experiences and Self-Perceptions			Relationship Between Self-Perceptions to Wanting to Learn More Mathematics			Relationship Between Mathematics Experiences, Self-Perceptions and Wanting to Learn More Mathematics		
	B	*SE B*	*β*	*B*	*SE B*	*β*	*B*	*SE B*	*β*
Fall (n = 299)									
Perceived opportunities to participate in school mathematics	0.12	0.03	0.21*				0.03	0.02	0.05
Mathematics self-perceptions				0.31	0.02	0.60*	0.31	0.02	0.59*
R^2			0.10			0.41			0.41
F			11.58*			171.67*			121.47*
Spring (n = 310)									
Perceived opportunities to participate in school mathematics	0.12	0.02	0.21*	0.30	0.02	0.63*	0.02	0.02	0.04
Mathematics self-perceptions							0.29	0.02	0.62*
R^2			0.07			0.42			0.42
F			32.74*			138.62*			99.28*

*$p < .001$.

opportunities to participate in mathematics and interest in mathematics was not significant, which suggests that mathematics self-perceptions plays an important role in mediating the effect of opportunities to participate in mathematics activities and interest in mathematics.

In order to account for indirect effects, an approach proposed by Sobel (1982), 1986; see also Bauer, Preacher, & Gill, 2006; Kenny, Korchmaros, & Bolger, 2003; Krull & MacKinnon, 1999, Krull & MacKinnon (2001); MacKinnon, 2008; MacKinnon, Lockwood, & Williams, 2004; MacKinnon, Warsi, & Dwyer, 1995) indicated statistically significant effects for the fall (Sobel's z = 3.69, $p < .001$; magnitude of indirect effect = 0.07, standard error = 0.02) and spring, (Sobel's z = 4.36, $p < .001$; magnitude of indirect effect = 0.09, standard error = 0.02). In addition, bootstrap confidence intervals for the indirect effects of 20,000 bootstrapped samples (Preacher & Hayes, 2008) did not include zero for the fall (CI [0.03, 0.12]) or spring (CI [0.05, 0.13]). Taken together, these results suggest a non-zero indirect effect for both the fall and spring data. That is, it appears that opportunities to participate in mathematics indirectly affect interest in learning more math through student mathematics self-perceptions.

Discussion

The results highlight the importance of students' self-perceptions in understanding the effects of mathematics learning opportunities on their desire to learn more mathematics. School mathematics opportunities are important, but alone may not be sufficient to increasing student interest in learning more mathematics (Greer, Verschaffel, & Mukhopadhyay, 2007; Jones, Uribe-Florez, & Wilkins, 2011; Ma & Kishor, 1997; NCTM, 2000; Wang, 2012). This has notable implications for teachers because teachers influence both school mathematics activities and students' sense of self as a mathematics learner. The results of this study provide quantitative evidence that students' desire to learn more mathematics in the future is a function of not only the extent to which they have classroom opportunities to learn math, but also whether they develop positive perceptions of themselves in mathematics (Schunk & Richardson, 2011).

The data show that students' mathematics self-perceptions played a critical role in mediating the effects of mathematics learning opportunities on their desire to learn more mathematics. Thus, school mathematics opportunities may be necessary, but not sufficient to increasing student interest in learning more about mathematics (Meece, Anderman, & Anderman, 2006; Pintrich, 2003). Supporting the desire to learn more mathematics is not simply about providing more of particular types of mathematics opportunities. It also requires paying attention to whether the activities and their implementation allow and encourage students to perceive themselves as the kind of person who can and wants to do mathematics. This finding may help guide teacher-student interactions as well as classroom activity development.

Although classroom and school demographic characteristics were not considered, the relatively small number of classrooms and schools prevented us from controlling for classroom or school level differences beyond statistically accounting for the nested structure of students within classrooms. One hypothesis is that classroom differences contribute to both the opportunities to learn mathematics and self-perceptions due to a non-random way in which students may have been assigned to their classrooms (over which we had no control of in this study). However, such classroom differences could also reflect different instructional practices and affordances by particular teachers, a possibility consistent with previous research (see for example, Battey, 2013; Bostic, Pape, & Jacobbe, 2016; Moyer, Robinson, & Cai, 2018). Our study suggests two ways teachers may be instrumental in students' learning ambitions. Teachers have considerable choice in which parts of the intended curriculum they enact (which units, specific lessons, and activities within lessons) and how they do so, thus shaping students' opportunities to participate in desirable mathematics activities. In addition, teachers can also influence the way students perceive themselves when engaged in learning mathematics. One implication of our results is that teacher preparation and professional development efforts might be more effective when informed by social practice theory and identity development. Specifically, teachers may need explicit support in why and how to help all students not just to access important mathematics learning activities but also to develop positive self-perceptions as mathematics learners (Blazer & Kraft, 2017). Teachers may not realize that student interest and persistence in mathematics reflects what they believe their teachers think of their capacity to learn it (Boaler, 2016). Another limitation of this study was the use of student self-reported data. We were not able to collect additional information through other data sources such observations or interviews (see for example, Berger & Karabenick, 2016) so future studies should include methods to assess the validity of inferences.

Overall, our results reiterate the importance of earlier mathematics education as a critical influence on whether students want to continue learning mathematics in the future. This study also provides important empirical evidence to support improving, providing, and conducting further research about broad access to not only reform-based learning opportunities but also the identity-forming aspects of mathematics education for all students in order to encourage more students to want to learn mathematics. To further this goal, the continued development of rigorous tools and methods for assessing the learning opportunities afforded students in mathematics as well as their emerging sense of themselves as capable mathematics learners is encouraged. In the development of these tools, the field must also consider what validity evidence is warranted for desired score interpretations and how that evidence is collected (Bostic et al., 2019).

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Jeffrey Shih http://orcid.org/0000-0002-4648-3963
Marsha Ing http://orcid.org/0000-0002-4156-8239

References

Atkinson, J. W. (1957). Motivational determinants of risk taking behavior. *Psychological Review*, *64*(6,Pt. 1), 359–372.

Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavior change. *Psychological Review*, *84*(2), 191–215.

Battey, D. (2013). "Good" mathematics teaching for students of color and those in poverty: The importance of relational interactions in instruction. *Educational Studies in Mathematics*, *82*(1), 125–144. doi:10.1007/s10649-012-9412-z

Bauer, D. J., Preacher, K. J., & Gil, K. M. (2006). Conceptualizing and testing random indirect effects and moderated mediation in multilevel models: New procedures and recommendations. *Psychological Methods*, *11*(2), 142–163. doi:10.1037/1082-989X.11.2.142

Berger, J., & Karabenick, S. A. (2016). Construct validity of self-reported metacognitive learning strategies. *Educational Assessment*, *21*(1), 19–33. doi:10.1080/10627197.2015.1127751

Blazer, D., & Kraft, M. A. (2017). Teacher and teaching effects on students' attitudes and behaviors. *Educational Evaluation and Policy Analysis*, *39*(1), 146–170. doi:10.3102/0162373716670260

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass & Pfeiffer Imprints.

Bobis, J., Anderson, J., Martin, A., & Way, J. (2011). A model for mathematics instruction to enhance student motivation and engagement. In D. Brahier & W. Speer (Eds.), *Motivation and disposition: Pathways to learning mathematics* (pp. 31–42). Reston, VA: National Council of Teachers of Mathematics.

Bostic, J., Krupa, E., & Shih, J. J., Bostic, E. Krupa, & J. Shih(2019). Introduction: Aims and scope for assessments in mathematics education contexts: Theoretical frameworks and new directions. In J. Bostic, E. Krupa, & J. Shih (Eds.), *Quantitative measures of mathematical knowledge: researching instruments and perspectives*. New York, NY: Routledge.

Bostic, J. D., Pape, S. J., & Jacobbe, T. (2016). Encouraging sixth-grade students' problem-solving performance by teaching through problem solving. *Investigations in Mathematics Learning*, *8*(3), 30–58. doi:10.1080/24727466.2016.11790353

Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19–33). Mahwah, NJ: Lawrence Erlbaum.

Collaboration, O. S. (2012). An open, large-scale collaborative effort to estimate the reproducibility of psychological science. *Perspectives on Psychological Science*, *7*(6), 657–660. doi:10.1177/1745691612462588

Desimone, L. M, Smith, T. M, & Frisvold, D. E. (2010). Survey measures of classroom instruction: Comparing student and teacher reports. *Educational Policy*, *24*(2), 267–329. doi:10.1177/0895904808330173

Donovan, M. S., Bransford, J. D., & Pellegrino, J. W. (Eds.). (1999). *How people learn: Bridging research and practice.* Washington, DC: National Academy Press.

Eccles, J. S., Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J. L., & Midgley, C. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motivation* (pp. 74–146). San Francisco, CA: W. H. Freeman.

Eccles, J. S., & Wang, M. (2015). What motivates females and males to pursue careers in mathematics and science? *International Journal of Behavioral Development*, *40*(2), 100–106. doi:10.1177/0165025415616201

Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology*, *53*, 109–132. doi:10.1146/annurev.psych.53.100901.135153

Gaspard, H., Dicke, A., Flunger, B., Brisson, B. M., Häfner, I., Nagengast, B., & Trautwein, U. (2015). Fostering adolescents' value beliefs for mathematics with a relevance intervention in the classroom. *Developmental Psychology*, *51*(9), 1226–1240. doi:10.1037/dev0000028

Gojak, L. M. (2013). *Algebra: Not 'if' but 'when'*. NCTM Summing Up. Washington, DC: National Council of Teachers of Mathematics.

Greer, B., Verschaffel, L., & Mukhopadhyay, S. (2007). Modeling for life: Mathematics and children's experience. In W. Blum, P. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education* (pp. 89–98). New York, NY: Springer.

Harel, G., Selden, A., & Selden, J. (2006). Advanced mathematical Thinking. In A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education past, present and future* (pp. 147–172). Rotterdam, Netherlands: Sense Publishers.

Jones, B., Uribe-Florez, L., & Wilkins, J. (2011). Motivation and disposition: The hidden curriculum. In D. Brahier & W. Speer (Eds.), *Motivation and disposition: Pathways to learning mathematics* (pp. 215–227). Reston, VA: National Council of Teachers of Mathematics.

Kenny, D. A., Korchmaros, J. D., & Bolger, N. (2003). Lower level mediation in multilevel models. *Psychological Methods*, *8*(2), 115–128.

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equality and variable. *Zentralblatt Für Didaktik Der Mathematik - International Reviews on Mathematical Education*, *37*, 68–76. doi:10.1007/BF02655899

Krull, J. L., & MacKinnon, D. P. (1999). Multilevel mediation modeling in group-based intervention studies. *Evaluation Review*, *23*(4), 418–444. doi:10.1177/0193841X9902300404

Krull, J. L., & MacKinnon, D. P. (2001). Multilevel modeling of individual and group level mediated effects. *Multivariate Behavioral Research*, *36*(2), 249–277. doi:10.1207/S15327906MBR3602_06

Ma, X., & Kishor, N. (1997). Attitudes towards self, social factors and achievement in mathematics: A meta-analytic review. *Educational Psychology Review*, *9*(2), 89–120. doi:10.1023/A:1024785812050

MacKinnon, D. P. (2008). *Introduction to statistical mediation analysis*. Mahwah, NJ: Erlbaum.

MacKinnon, D. P., Lockwood, C. M., & Williams, J. (2004). Confidence limits for the indirect effect: Distribution of the product and resampling methods. *Multivariate Behavioral Research*, *39*(1), 99–128. doi:10.1207/s15327906mbr3901_4

MacKinnon, D. P., Warsi, G., & Dwyer, J. H. (1995). A simulation study of mediated effect measures. *Multivariate Behavioral Research*, *30*(1), 41–62. doi:10.1207/s15327906mbr3001_3

Meece, J. L., Anderman, E. M., & Anderman, L. H. (2006). Classroom goal structure, student motivation, and academic achievement. *Annual Review of Psychology*, *57*, 487–503. doi:10.1146/annurev.psych.56.091103.070258

Moyer, J. C., Robinson, V., & Cai, J. (2018). Attitudes of high-school students using traditional and reform mathematics curricula in middle school: A retrospective analysis. *Educational Studies in Mathematics*, *98*(2), 115–134. doi:10.1007/s10649-018-9809-4

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Jacquelynne, S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation*, *21*(4), 343–370. doi:10.1080/13803611.2015.1057161

Nagy, G., Trautwein, U., Baumert, J., Köller, O., & Garret, J. (2006). Gender and course selection in upper secondary education: Effects of academic self-concept and intrinsic value. *Educational Research and Evaluation*, *12*(4), 323–345. doi:10.1080/13803610600765687

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2007). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2016). *Principles to action: Engaging mathematics success for all.* Reston, VA: Author.
National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards mathematics*. Washington, DC: Author.
National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
Nosek, B. A., Alter, G., Banks, G. C., Borsboom, D., Bowman, S. D., Breckler, S. J., … Yarkoni, T. (2015). Promoting an open research culture. *Science*, *348*(6242), 1422–1425. doi:10.1126/science.aab2374
Open Science Collaboration. (2015). Estimating the reproducibility of psychological science. *Science*, *349*(6251), aac4716. doi:10.1126/science.aac4716
Pintrich, P. R. (2003). A motivational science perspective on the role of student motivation in learning and teaching contexts. *Journal of Educational Psychology*, *95*(4), 667–686. doi:10.1037/0022-0663.95.4.667
Pintrich, P. R., & Schrauben, B. (1992). Students' motivational beliefs and their cognitive engagement in classroom academic tasks. In D. H. Schunk & J. L. Meece (Eds.), *Student perceptions in the classroom* (pp. 149–183). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods*, *40*(3), 879–891.
Rotter, J. B. (1966). Generalized expectancies for internal versus external control of reinforcement. *Psychological Monographs: General and Applied*, *80*(1), 1–28. doi:10.1037/h0092976
Schunk, D., & Richardson, K. (2011). Motivation and self-efficacy in mathematics education. In D. Brahier & W. Speer (Eds.), *Motivation and disposition: Pathways to learning mathematics* (pp. 13–30). Reston, VA: National Council of Teachers of Mathematics.
Schunk, D. H. (1992). Theory and research on student perceptions in the classroom. In D. H. Schunk & J. L. Meece (Eds.), *Student perceptions in the classroom* (pp. 3–23). Hillsdale, NJ: Lawrence Erlbaum Associates.
Schunk, D. H., & Meece, J. L. (1992). *Student perceptions in the classroom*. Hillsdale, NJ: Lawrence Erlbaum Associates.
Schunk, D. H., Meece, J. L., & Pintrich, P. (2014). *Motivation in education: Theory, research and applications* (4th ed.). Columbus, OH: Pearson Merrill Prentice Hall.
Simpkins, S. D., Davis-Kean, P. E., & Eccles, J. S. (2006). Math and science motivations: A longitudinal examination of the links between choices and beliefs. *Developmental Psychology*, *42*(1), 70–83. doi:10.1037/0012-1649.42.1.70
Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *The Journal of Educational Research*, *95*(6), 323–332. doi:10.1080/00220670209596607
Skinner, E. A., Wellborn, J. G., & Connell, J. P. (1990). What it takes to do well in school and whether I've got it: A process model of perceived control and children's engagement and achievement in school. *Journal of Educational Psychology*, *82*(1), 22–32. doi:10.1037/0022-0663.82.1.22
Sobel, M. E. (1982). Asymptotic intervals for indirect effects in structural equations models. *Sociological Methodology*, *13*(1982), 290–312. doi:10.2307/270723
Sobel, M. E. (1986). Some new results on indirect effects and their standard errors in covariance structure models. *Sociological Methodology*, *16*(1986), 159–186. doi:10.2307/270922
Wang, M. T. (2012). Educational and career interests in math: A longitudinal examination of the links between classroom environment, motivational beliefs, and interests. *Developmental Psychology*, *48*(6), 1643–1657. doi:10.1037/a0027247
Weiner, B. (1990). History of motivational research in education. *Journal of Educational Psychology*, *82*(4), 616–622. doi:10.1037/0022-0663.82.4.616
Wigfield, A. (1994). Expectancy-value theory of achievement motivation: A developmental perspective. *Educational Psychology Review*, *6*, 49–78. doi:10.1007/BF02209024
Wigfield, A., & Eccles, J. (1992). The development of achievement task values: A theoretical analysis. *Developmental Review*, *12*, 265–310. doi:10.1016/0273-2297(92)90011-P
Wigfield, A., & Eccles, J. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology*, *25*, 68–81. doi:10.1006/ceps.1999.1015
Wilburne, J. M., & Dause, E. (2017). Teaching self-regulated learning strategies to low-achieving fourth-grade students to enhance their perseverance in mathematical problem solving. *Investigations in Mathematics Learning*, *9*(1), 38–52. doi:10.1080/19477503.2016.1245036
Wilkie, K. J., & Sullivan, P. (2018). Exploring intrinsic and extrinsic motivational aspects of middle school students' aspirations for their mathematics learning. *Educational Studies in Mathematics*, *97*(3), 235–254. doi:10.1007/s10649-017-9795-y