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First-Grade Basic Facts: An Investigation Into Teaching and Learning of an Accelerated, High-Demand Memorization Standard

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California is one of 4 states that have accelerated addition and subtraction basic-facts memorization. This article reports on teacher practices, first-grade achievement of the standard, and a broader conception of basic-facts competence. Even among students from the highest performing schools, fewer than 11% made progress toward the memorization standard equivalent to their progress through the school year. Several negative correlations between instructional strategies and student retrieval suggest that teachers may benefit from professional development targeted at basic-facts teaching and learning. Textbook reliance was negatively correlated with basic-facts retrieval, suggesting that educators and policymakers may want to reexamine assumptions about the efficacy of traditional first-grade textbooks. This study's findings may prove useful to teachers, professional development trainers, and textbook publishers as they consider ways to improve basic-facts learning among early elementary children.

Key words: Addition, subtraction; Arithmetic; Children's strategies; Early number learning; Number sense; Policy; Teaching effectiveness

The question of how and when students should become fluent with addition and subtraction facts such as 9 + 7 and 15 – 6 has been the subject of much attention over the years. Although Washburne and the Committee of Seven acknowledged that students could learn "harder" addition and "easy" subtraction facts at age 7–8, and "the more difficult subtraction facts" at age 8–9 (as cited in Ilg & Ames, 1951, pp. 16–17), the accepted practice until just a few years ago was to present addition and related subtraction facts to 10 or 12 for all students during first grade (6–7 years old), and to continue their memorization work with the larger basic facts during second grade (7–8 years old) and beyond (Ashcraft, 1990; Fox, 1995).

NCTM has recently recommended that students develop recall of the basic addition and subtraction facts by the end of second grade (2006, p. 14). At least three states (California, South Carolina, South Dakota) have accelerated this timeline to

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The research reported here is one focus of the first author's doctoral dissertation (Henry, 2004).

first grade and have restricted the definition of "knowing" to that of memorization/recall.[1]

California, educator of approximately one sixth of all U.S. students, has committed to world-class expectations by setting a standard that all students study algebra 1 in eighth grade (California State Board of Education, 1999). To this end, kindergarten through seventh grade standards accelerate the study of positive rational number arithmetic in order to begin the study of negative numbers and symbolic algebra in fourth grade. California's first-grade Number Sense standard reflects this acceleration, stating that students should "know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory" (California State Board of Education, 1999, First-Grade Number Sense Standard 2.1). This standard is not repeated in second grade; instead, California's second-grade students are now charged with what was previously a third-grade basic-facts expectation—to begin the memorization of their multiplication facts (2s, 5s, and 10s).

With such a dramatic change in expectations, questions arise about the ability of young students to achieve more, earlier. A number of studies have provided evidence that U.S. children seldom achieve memorization of all addition and subtraction facts in the early grades of school; more often, these studies have demonstrated a combination of methods that moves over time from a heavy reliance on counting (K–3) toward memorization in the upper-elementary grades (Ashcraft, 1990; Fox, 1995; Fuson, 1992). This article reports on a study conducted during the 2003–2004 school year that investigated the teaching of basic facts in a high-demand environment, and the actual achievement of first-grade students when a systemic focus on basic facts memorization is in force.

BACKGROUND

The differences in the timing and wording of various states' basic-facts standards point to fundamental questions about the ways in which children and adults solve addition and subtraction facts, and the implications of different instructional approaches for the short- and long-term mathematical achievement of children. Although there appears to be little disagreement that basic-facts fluency is important for all children, there are several competing views on what constitutes basic facts fluency and how best to help children achieve this fluency.

One prevalent theory of basic-facts learning posits that children strengthen the association between basic-fact problems and their answers through repeated practice, building stronger bonds that lead to confident retrieval from long-term memory (Ashcraft, 1995; Baroody, 2003; Fox, 1995; Geary, 1994; Groen & Parkman, 1972). Based on this strategy-choice model (Siegler & Jenkins, 1989), children who can accurately solve problems with counting strategies are able to engage in the repetitions required to strengthen the bonds of association. Instructional programs that

[1] A fourth state, Indiana, has established a first-grade standard more open to interpretation: Students are called on to "demonstrate mastery."

incorporate direct instruction and repeated practice to develop accurate and efficient counting strategies embody this theory of learning.

Several researchers have focused attention on the developmental trajectory that students tend to follow as they develop accurate and efficient counting strategies. Carpenter and Moser (1984) observed five levels of basic-facts problem-solving development in first through third graders: Level 0—students unable to solve any addition or subtraction problems; Level 1— students using direct modeling strategies (counting all and adding on with objects or fingers); Level 2—students using both modeling and verbal/mental counting strategies; Level 3— students relying primarily on verbal/mental counting strategies; Level 4—students using basic-facts knowledge (including retrieval and derived facts) to solve addition and subtraction problems. Carpenter and Moser suggested that most classroom instruction at that time did not support this developmental trajectory but instead jumped "directly from the characterization of addition and subtraction through simple physical models to the memorization of number facts without acknowledging that there is an extended period during which children count-on and count back to solve addition and subtraction problems" (Carpenter & Moser, 1984, p. 200).

More recent work (Baroody, 1999, 2003; Brownell, 1935; Fuson, 1992; Gray & Tall, 1994) has suggested that children who solve problems based on their developing understanding of counting are likely to build their understanding of number relationships and properties, and develop part-whole, or derived-fact, strategies that can be highly efficient in solving basic-fact problems (see Figure 1). These derived-fact strategies have the added advantage of providing children with tools to solve mental math problems with multidigit numbers.

Concerns have been raised, however, that children who take an instrumental approach to counting to solve basic-fact problems are unlikely to develop this rich sense of numbers and tend to develop a growing reliance on counting over time. In a study of 72 English students, ages 7 through 12, Gray (1991) found a clear difference between the approaches that below-average and average/above-average students[2] had developed to solve basic-fact problems they could not retrieve from long-term memory. Typically, above-average students were found to use part-whole, derived-fact "short-cuts" that enhanced their relational schema of numbers. Although not as quick to develop these deductive reasoning strategies, students that teachers identified as average mathematical learners were also developing part-whole, relational reasoning. The below-average students, however, used counting as their primary fall-back method to solve problems they could not remember. Gray found that these students did not progress from counting to more relational approaches, and consequently,

> the bits they do know do not appear to be held together, with the result that this change in strategy may involve the child in long sequences of counting to arrive at solutions. In one sense they make things more difficult for themselves and as a consequence become less able. (pp. 569–570)

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[2] These designations were based on teacher identification prior to the start of student interviews.

	Addition	Subtraction
Counting	Direct modeling (counting objects and fingers) • Counting all • Counting on from first • Counting on from larger Counting abstractly • Counting all • Counting on from first • Counting on from larger	Counting objects • Separating from • Separating to • Adding on Counting fingers • Counting down • Counting up Counting abstractly • Counting down • Counting up
Reasoning	Properties • $a + 0 = a$ • a + 1 = next whole number • Commutative property Known-fact derivations (e.g., 5 + 6 = 5 + 5 + 1; 7 + 6 = 7 + 7 – 1) Redistributed derived facts (e.g., 7 + 5 = 7 + (3 + 2) = (7+ 3) + 2 = 10 + 2 = 12)	Properties • $a – 0 = a$ • a – 1 = previous whole number Inverses/complement of known addition facts (e.g., 12 – 5 is known because 5 + 7 = 12) Redistributed derived facts (e.g., 12 – 5 = (7 + 5) – 5 = 7 + (5 – 5) = 7)
Retrieval	Retrieval from long-term memory	Retrieval from long-term memory

Figure 1. Methods for solving basic-fact problems (see Baroody & Coslick, 1998; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Fuson, 1992; Fuson & Kwon, 1992; Kulm, 1985).

Not only are students who continue to rely on counting potentially deprived of opportunities to develop more robust number sense, they are also likely to face a growing problem as they tackle more complex mathematical problems. Research has provided mounting evidence of the correlation between strong basic-facts fluency and strong mathematics achievement (see, for example, Kilpatrick, Swafford, & Findell, 2001). From an information processing perspective, this correlation may be explained by noting that students who use less working memory to calculate the basic facts have more working memory available to apply those facts in more complex or novel situations (Bjorklund, Muir-Broaddus, & Schneider, 1990).

Although some mathematics educators advocate a developmental approach to basic-facts fluency, others (including the state of California) focus on an end product of memorization. However, evidence from “expert” children and adults indicates that successful arithmetic skills are often accomplished using a combination of memory and strategy techniques (Baroody, 1999; Bisanz & LeFevre, 1990; Campbell & Xue, 2001; Gray & Tall, 1994; Kilpatrick et al., 2001; LeFevre, Smith-Chant, Hiscock, Daley, & Morris, 2003). These strategy techniques (derived strategies) fall into at least two main categories (Fuson, 1992): (a) *redistributed* *derived facts* (for 7 + 5, a child might decompose 5 into 3 + 2, and then add 7 + 3 to get 10, and then add 2 onto the 10), and (b) *known fact derivations* (for 7 + 5, a child might recall that 5 + 5 = 10 and 2 more is 12).

Although California cited its interest in establishing world-class standards, it may have misinterpreted the educational practices of high-performing countries (based on results from TIMSS). Studies of educational practices in Korea, mainland China, Taiwan, and Japan have found that students are not simply drilled on basic facts using memorization-focused approaches. Instead, they are provided with explicit and sustained instruction on redistributed derived-fact strategies during first grade (Fuson & Kwon, 1992; Fuson, Stigler, & Bartsch, 1988):

1. Up-over-10, an addition strategy in which one addend is decomposed such that one part will combine with the other addend to make 10, and the second part of the decomposed addend is added on to the 10 (e.g., 8 + 7 = 8 + 2 + 5 = 10 + 5 = 15).
2. Down-over-10, a subtraction strategy in which the subtrahend is decomposed so that one of the two parts is the same as the amount over 10 in the minuend, and then the other part of the decomposed subtrahend is subtracted from the 10 (e.g., 15 – 7 = 15 – (5 + 2) = (15 – 5) – 2 = 10 – 2 = 8).
3. Take-from-10, a subtraction strategy in which the minuend is essentially decomposed into 10 and the remainder, the subtrahend, is then subtracted from 10, and finally the remainder of the decomposed minuend is added to the difference (e.g., 15 – 7 = (5 + 10) – 7 = 5 + (10 – 7) = 5 + 3 = 8).

Thus, it appears that children from several high-performing countries develop strong memorized facts in first grade for sums up to 10, and then develop a combination of memorized facts and recomposition strategies to solve sums and differences beyond ten (Peak, 1997). As Fuson and Kwon (1992) noted, even before formal first-grade instruction, counting strategies accounted for only 19% of the solutions for sums over 10.

Studies suggest that emphasizing strategic acquisition of basic facts has at least one key advantage over focusing on memorization: Students who learn to group by 5s and 10s using composition/decomposition strategies (e.g., 5 + 8 = 5 + [5 + 3] = [5 + 5] + 3 = 10 + 3 = 13) may be more likely to develop a base-10 understanding of numbers and regrouping than students who rely on memory and counting strategies (Cotter, 1996; Fuson, 1992; Miura et al., 1994).

In addition to targeting memorization of all addition facts to 20, California's first-grade standard also designates that same level of memorization for subtraction facts. For policymakers, it may seem logical to assume that anyone who can memorize the addition facts can just as easily memorize the subtraction facts. However, research has found that children do not find the complementary relationship between addition and subtraction obvious, particularly when their confidence with addition facts is still evolving (Baroody, 1999; Hiebert & Wearne, 1992). Young children also appear to have more difficulty learning their subtraction facts because they often have less facility counting down than they do counting up (Fuson, 1992). Without special attention, subtraction facts may continue to be more difficult than addition facts well into adulthood. LeFevre et al. (2003) found that competent adults used retrieval from long-term memory for subtraction fact problems much less frequently than for addition. In a study comparing Canadian and Chinese university students (Campbell & Xue, 2001), non-Asian Canadian students reported using retrieval for subtraction facts only 57% of the time (73% for smaller facts and 42% for larger facts). Although the percentages of reported retrieval were higher for both Chinese and Chinese-Canadian students (79% each), the retrieval rates for larger subtraction problems such as 13 – 5 and 13 – 8 were still relatively low (65% and 64%, respectively). When compared with the reported retrieval rates for addition (76%, 92%, and 95%, respectively), the subtraction rates provide evidence that even well-educated adults do not have equal mastery of the two operations.

With these issues in mind, this study sought to shed light on how first-grade students are doing relative to California's accelerated mathematics standard. Because state-mandated testing in California does not extend to first grade, little is known about the student achievement of these accelerated standards. Thus, this study focused on three key questions:

1. To what extent are California's first-grade students achieving the basic-facts standard?
2. What instructional strategies are related to student attainment of the standard?
3. How do various instructional decisions interact with student number sense?

It is important to note that by the 2003–2004 school year, teachers in this study had had 4 years to transition to these higher basic-facts expectations and were all in their 2nd or 3rd year of implementation of standards-aligned textbooks required by California's Board of Education.

METHODS

Participants

To address the interrelated teaching and learning questions posed above, both teachers and students were surveyed in this study. To capture potential differences in instruction influenced by variations in school and district accountability systems and district-mandated textbooks, teachers and students were sampled across nine elementary sites located in four different school districts in southern California.

The nine schools represent a variety of statewide performance ranks,[3] student demographics, and textbook usage. Based on demographic data from California's 2003–2004 STAR data, the schools ranged from 3% to 94% economically disad-

[3] Statewide performance rank indicates the decile that each school falls into based on California's Academic Performance Index (API). The API ranges from 200 to 1000, and is designed to measure academic performance and progress at the school level.

vantaged (*M* = 50%), 8% to 92% English language learners (*M* = 47%), and 1% to 87% White (*M* = 42%). Based on the 2003 API scores, three of the schools were in the top decile of California elementary schools, two were from mid deciles (6 and 8), and three were from lower deciles (1, 2, and 3). The ninth school was in its 1st year of operation and did not have a 2003 API score. Although a district representative estimated that this new school might begin in the 8th decile, its first official API (2004) was in the 10th decile. Thus, instead of having three low-API, three mid-API, and three high-API schools in this study, the final ratio was actually 3:2:4. The nine schools were evenly divided in their use of three different state-approved textbooks (see Table 1): *California Mathematics* (Scott Foresman, 2001), *Mathematics* (Houghton Mifflin, 2002), and *McGraw-Hill Mathematics* (McGraw-Hill, 2002).

Table 1
School and District Participation Data

	District A	District B	District C	District D
Number of schools	3	3	2	1
Deciles	3, 6, 10	2, 10, 10	1, 8	10
Number of students interviewed	90	95	60	30
Textbook	Scott Foresman	Houghton Mifflin	McGraw-Hill	McGraw-Hill

Surveyed and Selected Teachers. All first-grade teachers from the nine schools were invited to complete surveys during April and May of 2004. Of the 55 eligible teachers, 49 (89%) completed the surveys. Of the 55 eligible teachers, 52 elected to administer untimed addition and subtraction basic-facts pretests to their students during the first 6 weeks of the 2003–2004 school year. From these 52 teachers, 3 teachers per school (Selected Teachers) were randomly chosen to provide students for assessments near the end of the year. Because of teacher interest and student availability, one school had four Selected Teachers, making a total of 28 Selected Teachers.

Interviewed students. Stratified sampling techniques were used to select 275 first-grade students from the nine school sites, who were interviewed during May and early June of 2004. These students were selected using the following protocol: Students from the Selected Teachers' classrooms were ranked according to the sums of their scores on the basic-facts pretests. Starting with the second-lowest-scoring child and skipping every other student until the top-scoring student had been reached, 10 students were pre-identified to be interviewed.[4] The remaining students were paired on the interview lists with the student who was the next closest in score; when a student was absent on the day of the scheduled interviews, the paired student was then interviewed. In cases in which a classroom had fewer than 20

[4] All of the schools in this study maintained first-grade classes of no more than 20 students each.

students, random selection methods were used to complete the list of 10 pre-identified students.

In eight of the nine schools, three sets of 10 students were interviewed.[5] In the ninth school (API rank = 10), due to teacher requests and student availability, two sets of 10 students, one set of 8, and one set of 7 (from four Selected Teachers) were interviewed, thus resulting in the number of students reported in Table 1. The students, on average, were 6.65 years old ($SD = 0.55$), and 54% of the sample was girls. Forty-seven percent of the students reported speaking English as a second language.[6]

Data Collection Instruments and Procedures

Three types of data collection instruments and procedures were used during this study: teacher surveys, addition and subtraction basic-facts pretests, and one-on-one assessment interview forms.

Teacher surveys. A 16-question teacher survey was personally administered to all regular education teachers teaching first graders at the participating schools during half-hour meetings at the school sites in April and May of 2004. Forty-five teachers completed the survey during these meetings; four teachers completed the surveys after the teacher meetings. The surveys provided data for each of the following variables:

1. Teacher experience
2. Teacher beliefs about various aspects of the basic-facts standard, as well as the first-grade place value standard
3. Levels of textbook and student workbook implementation, and implementation levels of supplemental activities that focused on basic facts. A 10-point scale for Textbook Reliance was used in correlational analyses and was based on teacher responses to two questions: (a) What percentage of your mathematics lessons generally follow the lesson suggestions from the textbook? (b) What percentage of the mathematics workbook pages from your student textbook have you had your students do so far (to this point in the year)? On each question, teachers who reported 0% to 20% scored 1 point, 21% to 40% scored 2 points, 41% to 60% scored 3 points, 61% to 80% scored 4 points, and 81% to 100% scored 5 points. In some analyses, teachers who reported 80% to 100% on both questions were coded as having the highest level of textbook reliance; all others were coded as less reliant.
4. Number of instructional events per week or month designed to help students learn the addition and subtraction facts, including pre-identified choices of (a) basic-facts worksheets, (b) flash cards, (c) timed tests, (d) mathematics games, (e)

[5] Parents of all students were notified via an Information Letter sent home by the classroom teachers, and were invited to contact the researcher or the classroom teacher to opt out of the interviews. One parent chose to exercise this option.

[6] Based on legal concerns, several schools declined to provide specific language proficiency levels.

solving word problems, (f) songs, (g) mathematics manipulatives, (h) fact family activities and worksheets, and optional fill-ins for "other." Teachers were also asked to identify their instructional focus for each type of instructional event as (a) developing noncounting strategies to find answers (e.g., making 10s, doubles plus 1), (b) getting the right answer (primarily by helping them develop quick and accurate counting strategies), and (c) memorizing the facts. Explanations for these three instructional foci were provided at the time the surveys were administered, and the surveyed teachers verbally indicated that they understood the distinctions among the three foci.

Basic-facts pretests. All first-grade students (with the exception of approximately 60 students from the classrooms of the three nonparticipating teachers) were pretested on both their addition and subtraction facts during the first 6 weeks of school. Each test contained 36 items selected from 2 + 2 to 9 + 9 and 18 – 9 to 4 – 2, and was administered in an untimed environment. In all cases, the first author completed the scoring of the tests.

Mathematics interview assessments. During May and early June 2004, the 275 preidentified students were asked to solve, and provide information about how they solved, 10 addition and 8 subtraction basic-fact problems (see Figures 2 and 3).[7] Students were asked each problem verbally, and also were able to read each problem from an assessment booklet that presented each problem on its own page in an invariant order, working first with half of the addition problems, then half of the subtraction, followed by the remaining half of the addition and then the last half of the subtraction problems. In most cases, students solved all basic-fact problems without access to manipulatives. In a few cases, students asked for manipulatives or number lines, and they were provided.

After answering each problem, students were asked to self-report on the method they used to solve the problem: (a) variations on counting, (b) derived-fact strategies, or (c) retrieval from long-term memory. In cases where students were clearly counting with fingers or counting verbally, this was coded without querying the students. In all other cases, the student was asked, "How did you get that?" Typical responses included "I just knew it"; "I counted in my head"; and "I knew 5 + 5 was 10, so 4 + 5 is 9." If a student gave a very quick answer but reported a nonmemory method or took extended time before answering but reported a memory method, the interviewer asked a gentle follow-up probe. In all cases, the students' self-reports were accepted and coded after these probes.

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[7] Facts were selected based on the following criteria: (a) plus/minus 0 and 1 facts were not included because first-grade students in a pilot study were generally unable to determine whether they solved these problems using retrieval from long-term memory or using a rule-based strategy; (b) facts that are designated by the California Math Standards for memorization during kindergarten were not selected for this first-grade assessment; (c) two addition facts each were selected for under-10 combinations, 10 combinations, and doubles, and four additional over-10 addition facts included two plus-8s and two plus-9s; and (d) eight subtraction facts included one each from the 8, 9, 10, 11, 12, 13, 14, and 15-minus facts, including one double that was the inverse of the 7 + 7 fact included in the addition items.

+	2	3	4	5	6	7	8	9
2	K							
3	K	K						
4								
5			X					
6	X		X		X			
7		X				X		
8		X				X		
9				X			X	

Figure 2. Distribution of assessed addition basic facts. Each assessed fact is marked with an X, facts targeted for memorization in kindergarten are marked with a K, and duplicate or out-of-range facts are shaded. All problems were posed with larger addend last.

–	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
2	K	K					X								
3		K													
4										X					
5					X										
6								X							
7						X					X				
8									X						
9												X			

Figure 3. Distribution of assessed subtraction basic facts (read across, then down). Each assessed fact is marked with an X, facts targeted for memorization in kindergarten are marked with a K, and duplicate or out-of-range facts are shaded.

This study was designed to assess student strategies for solving basic-fact problems based on self-reports, which have been shown to discriminate among retrieval, counting, and derived-fact strategies more reliably than the alternative method of timing students' responses and making inferences based on the speed with which they answer various problems (Hopkins & Lawson, 2002; LeFevre et al., 2003; Siegler & Jenkins, 1989). It is important to note, though, that some students made attempts to hide their use of finger counting. It is also possible that some students may have used derived-fact strategies but felt uncomfortable or unable to verbally describe them and would have defaulted to "I just knew it," which would then have been coded as retrieval. These factors suggest that self-reports of retrieval are likely to be inflated.

Four credentialed teachers not connected with any of the participating schools conducted the interviews, which generally lasted between 12 and 20 minutes.

Discussions were held after each set of interviews to clarify any coding questions. Responses were coded by the interviewers in two different ways: (a) accuracy of the response, and (b) solution method observed by the interviewer and/or reported by the student.

In addition to the 18 addition and subtraction problems, students were also asked 7 questions designed to assess their number sense: 3 word problems (Figure 4) and 2 related pairs of place-value questions (Figure 5). Coding again consisted of recording whether the answer was correct as well as students' self-reports about their

Word problem	Problem type
Maria has 7 dollars. She wants to buy a book that costs 11 dollars. How much more money does she need?	Join/change unknown 7 + ? = 11
Pete has 14 pencils. Six of his pencils are red, and the rest are blue. How many blue pencils does Pete have?	Part-whole/part unknown 6 + ? = 14
There were 24 children on the playground. Ten more children came out to play. How many children were on the playground then?	Join/result-unknown 24 + 10 = ?

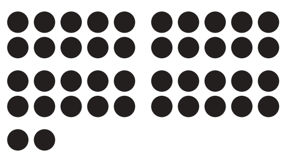
Figure 4. Mathematics assessment: word problems.

Directions for PV1 and PV2: Put out 3 bundles and 5 sticks, with bundles and sticks mixed spatially. Show the child that there are 10 sticks in each bundle. Count together the sticks in one of the bundles and assure the child that there are 10 sticks in each bundle.

PV1: HOW MANY STICKS ARE THERE ALTOGETHER? (Indicate the entire collection of sticks.)

PV2: If the child miscounts the set, count with him/her (by 1 if necessary to show that there are 35). Then cover the 35 and put out 2 more bundles. Tell the child that there are 10 in each bundle. WE HAVE 35 UNDER HERE, AND NOW I HAVE PUT OUT THESE. HOW MANY ARE THERE ALTOGETHER? (Move your hand over all the sticks including the covered sticks.) Repeat that there are 35 under the cover if necessary.

PV3: Show the child the card with four sets of 10 dots and 2 more dots. Be sure the child knows that each set of dots has 10 dots. HOW MANY DOTS ARE THERE ALTOGETHER?



PV4: Cover up one group of 10 dots. IF I COVER UP ONE GROUP OF 10, NOW HOW MANY DOTS CAN YOU SEE?

Figure 5. Mathematics assessment: place-value questions. Uppercase words indicate scripted interview questions.

methods of solving the problems (e.g., for Place Value Question 1, counting by 1s, counting by 10s, knowing that three 10s is 30 and 5 is 35).

These seven questions were combined to form a 7-point Number Sense Proficiency scale, which demonstrated acceptable psychometric qualities based on a scale reliability analysis, $\alpha = .72$. Each word problem that a student answered correctly, using any solution method, added 1 point to the scale score. Place-value problems were scored 0 if students used counting by 1s or 10s; they were scored 1 if students said they "just knew it" or gave an explanation that indicated they had used the number of groups of 10 to directly determine the number in the 10s place.

RESULTS

Student Attainment of Basic-facts Fluency

More than two thirds of the assessed first-grade students in these nine diverse schools were still using counting as their primary method of solving the 18 basic-fact problems with just a few weeks left in the school year (see Table 2). Students reported using retrieval from long-term memory (Basic Facts Retrieval) to solve a median of 22.22% of the addition and subtraction facts (M = 31.25, SD = 25.88).[8] These low results are particularly striking because students from high-performing schools were overrepresented, and the data collection protocols favored an overreporting of retrieval. Even extending the definition of Basic Facts Competence to include derived-fact strategies as well as retrieval, the median rose only slightly to 33.33% (M = 37.33, SD = 29.45).

Table 2
Performance Bands Summary: Retrieval and Basic Facts Competence

Student outcome measure	Median % of facts solved using retrieval (retrieval + derived-fact strategies median)	Performance bands (% of 275 students performing at or above)		
		Moderate: 50% or more retrieval (retrieval + derived-fact strategies)	Adequate: 80% or more retrieval (retrieval + derived-fact strategies)	Mastery: 100% retrieval (retrieval + derived-fact strategies)
Addition facts (n = 10)	30.00% (40.00%)	33.5% (45.0%)	12.0% (21.4%)	2.9% (8.7%)
Subtraction facts (n = 8)	12.50% (12.50%)	21.8% (27.3%)	7.6% (9.5%)	3.6% (5.1%)
Addition and subtraction (n = 18)	22.22% (33.33%)	25.5% (32.7%)	6.9% (12.8%)	2.2% (4.4%)

Note. These performance bands are overlapping rather than additive.

[8] Because student achievement data are not normally distributed, medians are reported as measures that are less influenced by skewed data.

Although such weak attainment of basic-facts fluency near the end of first grade has been reported previously, these results are noteworthy. Even when students in the highest-performing schools are considered separately, 53% relied on counting to solve at least half of the basic-fact problems. Among this group of students, Basic-Facts Competence was only marginally higher than for all students. The median for problems solved using either retrieval or derived-fact strategies was 38.89% (*M* = 45.24, *SD* = 30.78).

Even though the state of California has set the same first-grade basic-facts standard for both addition and subtraction, student achievement of the standard was markedly weaker for subtraction than for addition (Subtraction: *Mdn* = 12.50%, *M* = 16.55, *SD* = 23.28; Addition: *Mdn* = 30.00%, *M* = 35.93, *SD* = 28.12). The difference becomes even more pronounced when derived-fact strategies are included with retrieval to measure Basic-facts Competence (Subtraction: *Mdn* = 12.50%, *M* = 28.64, *SD* = 31.14; Addition: *Mdn* = 40.00%, *M* = 44.29, *SD* = 31.13).

Basic-Facts Item Analysis

With the exception of the doubles facts, students demonstrated a consistent pattern of higher retrieval of lower-sum facts than of higher-sum facts (Tables 3 and 4). When the percentages of students solving each fact by retrieval or derived-fact strategies are combined, two changes in relative ranking stand out. First, students used retrieval and derived-facts strategies least with 4 + 6, indicating that more students counted to solve this 10 fact than any other, including the +8 and +9 facts; second, students needed to count more frequently with 7 + 8 than with 8 + 9 despite its smaller sum.

Table 3
Ranking of Addition Basic Facts: Retrieval and Derived-Fact Strategies

	Retrieval: percentage of students solving		Derived-fact strategies: percentage of students solving		Sum of accurate retrieval and accurate derived-fact strategies
Facts	Correctly (rank)	Incorrectly	Correctly (rank)	Incorrectly	
6 + 6	74.2% (1)	2.2%	8.4% (5)	0.4%	82.6% (1)
7 + 7	50.5% (2)	2.5%	8.7% (4)	2.2%	59.2% (2)
4 + 5	49.5% (3)	1.1%	8.4% (5)	0.7%	57.9% (3)
2 + 6	43.6% (4)	1.1%	2.9% (8)	0.0%	46.5% (4)
3 + 7	37.1% (5)	2.2%	2.5% (9)	0.0%	39.6% (5)
3 + 8	30.5% (6)	1.5%	6.2% (6)	0.0%	36.7% (6)
4 + 6	18.7% (7)	1.5%	5.8% (7)	0.0%	24.5% (10)
5 + 9	16.4% (8)	2.5%	15.6% (1)	0.7%	32.0% (7)
7 + 8	15.3% (9)	2.2%	10.2% (3)	1.1%	25.5% (9)
8 + 9	13.8% (10)	2.9%	15.3% (2)	2.2%	29.1% (8)

Table 4
Ranking of Subtraction Basic Facts: Retrieval and Derived-Fact Strategies

Facts	Retrieval: percentage of students solving		Derived-fact strategies: percentage of students solving		Sum of accurate retrieval and accurate derived-fact strategies (rank)
	Correctly (rank)	Incorrectly	Correctly (rank)	Incorrectly	
10 – 2	45.8% (1)	1.8%	1.5% (5)	0.0%	47.3% (1)
8 – 5	37.5% (2)	0.4%	1.5% (5)	0.0%	39.0% (3)
14 – 7	37.5% (2)	0.4%	2.2% (4)	1.1%	39.7% (2)
9 – 7	20.8% (3)	1.8%	0.7% (6)	0.4%	21.5% (6)
11 – 6	20.3% (4)	1.5%	6.6% (1)	1.1%	26.9% (4)
13 – 4	16.1% (5)	1.8%	6.6% (1)	1.1%	22.7% (5)
12 – 8	13.6% (6)	1.1%	2.9% (3)	0.0%	16.5% (8)
15 – 9	12.9% (7)	3.7%	3.7% (2)	2.2%	16.6% (7)

Textbooks, Teaching, and Student Achievement

Using teacher survey data in combination with student achievement data, several relationships among textbook reliance, instructional practices, and student achievement have been identified. These findings raise important questions about the alignment of state-adopted instructional materials with state standards and also about the instructional events and approaches that teachers choose to enhance state-mandated materials.

Textbook reliance. In an effort to improve students' educational outcomes in mathematics, the state of California over the past few years has structured district-level incentives and penalties to ensure that all students have access to current state-approved textbooks. Professional development resources have been heavily channeled into workshops focused on effective use of these textbooks. State-level policymakers appear to have been operating under the assumption that high textbook reliance will lead to effective instruction and proficient levels of student achievement.

The results of this study do not confirm this assumption: Of the sampled students, 43% were taught by teachers who reported the highest level of textbook reliance (81% to 100% of mathematics lessons following textbook recommendations and assigning students 81% to 100% of textbook workbook pages). For these students, the median retrieval score on the combined basic facts was 16.67% (M = 25.66, SD = 22.28). The median basic-facts competence score (retrieval and derived-fact strategies) was 22.22% (M = 30.23, SD = 25.06). These statistics alone demonstrate that for these students, at least, the state-approved textbook programs were not effective in helping them meet the basic-facts standard.

Students experiencing lower levels of textbook reliance actually made better progress toward the standard than students experiencing the highest level of textbook reliance (see Table 5). Across all three performance levels, more students were successful when their teachers relied less heavily on the state-approved textbooks (for retrieval, $r = -.14$, $p < .05$; for retrieval plus derived strategies, $r = -.15$, $p = .01$). Higher textbook reliance actually demonstrated a positive correlation with student use of counting to solve basic-facts problems ($r = .14$, $p < .05$).

Table 5
Retrieval [*and Fluency*] *Performance Bands by Textbook Reliance*

	Performance levels (% of students performing at or above)		
Textbook reliance	Moderate (50% or more retrieval)	Adequate (80% or more retrieval)	Mastery (100% retrieval)
Highest[a]	18.6 [22.9]	1.7 [5.9]	0.8 [0.8]
Lower[b]	30.6 [40.1]	10.8 [17.8]	3.2 [7.0]

Note. Values in brackets include both retrieval and derived-fact strategies; those above the brackets include only retrieval from long-term memory. Retrieval is based on retrieval of all 18 addition and subtraction basic-fact problems. These performance bands are overlapping rather than additive.
[a]43% of the sampled students experienced the highest level of textbook reliance.
[b]57% of the sampled students experienced lower levels of textbook reliance.

Instructional events. With only one exception, the 28 Selected Teachers in this study reported using mathematical resources supplementary to the state-adopted instructional materials either occasionally (14) or frequently (13). When asked to quantify the average number of times per week they had used various basic-fact-related instructional activities since January, the Selected Teachers indicated an average of 15 additional activities each week ($SD = 5.06$, $Mdn = 14.50$). Many of the Selected Teachers, although not all, repeated their responses for subtraction, and often commented that they incorporated both addition and subtraction in the same instructional activities.

Although one might assume that teachers incorporated additional instructional events into the mathematics curriculum in order to help students achieve the state standard, that goal did not materialize in this study. No statistically significant correlations were found for the total number of instructional events that teachers reported and basic-fact retrieval ($r = .10$, $p = .10$ for addition; $r = .09$, $p = .13$ for subtraction). If the intent was, on the other hand, to help students develop basic-facts competence from a broader perspective of retrieval and derived-fact strategies, the total number of instructional events was significantly correlated with student achievement ($r = .19$, $p < .01$ for addition; $r = .18$, $p < .01$ for subtraction).

Throughout this study, teachers specifically asked for information about which types of instructional strategies were more effective in helping students reach California's first-grade basic-facts standard. For many, the question was whether to emphasize worksheets, flash cards, and timed tests, or whether to invest more time in hands-on activities, games, solving word problems,[9] and other types of activities. Only one statistically significant positive correlation was found between the use of specific instructional activities and Addition Facts Retrieval, and that was the use of basic-facts worksheets ($r = .16$, $p < .01$).[10] The correlation for Addition Facts Competence was slightly higher ($r = .20$, $p < .01$).

At least two of the participating schools instituted daily timed tests during the study year and are representative of teachers and schools looking for better ways to help their students meet the basic-facts standard. A stepwise regression looking at the contributions of students' pretest scores and the instructional contributions of basic-facts worksheets, flash cards, and timed tests, confirms the positive, albeit slight, predictive value, of worksheets on predicting Addition Facts Retrieval (see Table 6). Flash card use during school hours was not found to be predictive of student memorization of the addition facts, and increased use of timed tests negatively predicted student memorization. It is not surprising that the largest predictor of Addition Facts Retrieval was student pretest scores.

Table 6
Linear Regression Coefficients: Percent Addition Facts Retrieval Regressed on Pretests, Basic-Facts Worksheets, Flash Cards, and Timed Tests

		Unstandardized coefficients		Standardized coefficients	t	Sig.	95% confidence interval for β	
	R^2 change	β	Std. error	β			Lower bound	Upper bound
(Constant)	—	15.14	3.43	—	4.42	.00	8.40	21.89
Pretests	.15	.57	.08	.40	7.27	.00	.42	.73
Basic-facts worksheets	.02	3.60	1.05	.22	3.44	.00	1.54	5.67
Timed tests	.02	–2.37	1.06	–.14	–2.23	.03	–4.46	–.28

Note. Model 3 was used, $F(3, 271) = 20.88$, $p < .01$; $R^2 = .18$.

For Addition Facts Competence, similar results were obtained. However, the regression estimate for worksheets was slightly higher (see Table 7). As with Addition Facts Retrieval, timed tests were found to negatively predict Basic-Facts Competence, and the greatest predictor was student pretest scores.

[9] None of the teachers reported in this study indicated a use of word-problem-based instruction, such as that found in classrooms taught by teachers involved in Cognitively Guided Instruction (CGI) professional development. Some CGI studies have reported improved recall of number facts for students experiencing this type of problem-based instruction relative to students receiving more traditional instruction (Carpenter, Fennema, Franke, Levi, & Empson, 2000).

[10] A similar analysis for subtraction is not included herein for two major reasons: (1) there is less variance in the subtraction scores than in addition, and (2) many teachers were unable to distinguish instructional activities for addition and subtraction.

Table 7
Linear Regression Coefficients: Percent Addition Facts Competence Regressed on Pretests, Basic-Facts Worksheets, Flash Cards, and Timed Tests

Model		Unstandardized coefficients R^2 change	β	Std. error	Standardized coefficients β	t	Sig.	95% confidence interval for β Lower bound	Upper bound
3	(Constant)	—	14.20	3.48	—	4.08	.00	7.35	21.05
	Pretests	.27	.84	.08	.53	10.43	.00	.68	.99
	Basic-facts worksheets	.03	4.75	1.07	.26	4.46	.00	2.65	6.84
	Timed tests	.02	–2.92	1.08	–.16	–2.71	.01	–5.04	–.80

Note. Model 3 was used, $F(3, 271) = 41.81$, $p < .01$; $R^2 = .32$.

Although many teachers have relied on flash cards and timed tests to help students learn their basic facts, this study does not confirm the effectiveness of these strategies. Even though the data demonstrate a relatively small correlation between basic-fact worksheet use and retrieval, the strength of this association suggests that reliance on this instructional practice did not dramatically contribute to student basic-fact retrieval or fluency.

Instructional goals. Neither textbook reliance nor the incorporation of additional instructional events related to strong student achievement of the basic-facts standard in this study. Even when teachers said they were using the same types of instructional activity, it did not always follow that they implemented them in the same ways. Students' experiences and learning outcomes may be very different, depending in large part on their teachers' instructional goals. Teachers in this study attributed different learning goals to the same activities: (a) helping students get the correct answer, (b) helping students learn how to solve problems using derived-fact strategies, and (c) helping students memorize their facts.

From the stepwise linear regressions that include pretest scores and the total number of instructional events focused on three different learning goals, it appears that many instructional events focused on memorization may not be hitting the mark intended by the teachers. Over the course of any given week, teachers reported many instructional events focused on the three goals (see Table 8). Memory-focused instructional events did not significantly predict memorization of addition facts ($\beta = .01$, $t(274) = .25$, $p = .80$). In fact, a linear regression including pretests and all three categories of instructional events identified students' pretest scores as the only significant predictor of memorization ($\beta = .39$, $t(274) = 6.98$, $p < .01$, $R^2 = .15$, $F(1, 273) = 48.67$, $p < .01$).

The events teachers reported that focused on memorization did, however, slightly predict decreases in students' use of counting ($\beta = –.11$, R^2 change = .01 out of an $R^2 = .18$ for the model, $p < .05$; $F(2, 272) = 30.13$, $p < .01$ for the model).

Table 8
Average Number of Basic Facts Instructional Events Per Week by Instructional Goal: Selected Teachers (n = 28)

	Addition		Subtraction	
Instructional goal	Mean (*SD*)	Median	Mean (*SD*)	Median
Memorization	3.45 (3.47)	1.75	3.21 (3.25)	1.25
Developing strategies for finding the answers	7.88 (5.21)	7.00	7.43 (5.25)	7.00
Getting the answer right	3.61 (2.92)	3.50	4.00 (3.40)	3.75

Instructional events focused on memorization (β = .27, R^2 change = .05 out of an R^2 = .19 for the model, $p < .01$; $F(3, 271) = 21.98$, $p < .01$ for the model) and on developing strategies (β = .26, R^2 change = .06 out of an R^2 = .19 for the model, $p < .01$; $F(3, 271) = 21.98$, $p < .01$ for the model) were equally predictive of increases in students' use of derived-fact strategies.

Under-10 Facts, Doubles, 10-Facts

Many of the students surveyed in this study indicated that their teachers emphasized memorization of addition doubles. Several teachers reported their belief that memorizing the doubles facts is an important prerequisite to memorizing or deriving the other facts. Even though doubles were the most frequently memorized facts (6 + 6 at 74.2% and 7 + 7 at 50.5%), they were not the primary predictor of memorization of the more difficult over-10 addition facts. A stepwise linear regression (Tables 9 and 10) identified the contributions of memorization of doubles (6 + 6 and 7 + 7), 10-facts (3 + 7, 4 + 6), and under-10 facts (2 + 6, 4 + 5) to the memorization of the four over-10 facts that were not doubles (3 + 8, 7 + 8, 5 + 9, 8 + 9). The two 10-facts contributed the largest R^2 of .36, with much smaller contributions from the under-10 facts and doubles facts. Thus, it would appear that memorization of doubles was less contributory to memorizing the more difficult combinations than the 10-fact couplets.

Table 9
Linear Regression Model Summary: 10-Facts, Under-10 Facts, and Doubles Facts as Predictors of Over-10 Facts

					Change statistics				
Model	*R*	R^2	Adjusted R^2	Std. error of the estimate	R^2 change	*F* change	*df*1	*df*2	Sig. *F* change
1[a]	.60	.36	.36	.92	.36	156.34	1	273	.00
2[b]	.64	.41	.41	.88	.05	21.57	1	272	.00
3[c,d]	.66	.43	.43	.87	.02	11.14	1	271	.00

[a]Predictors: (Constant), 10-facts
[b]Predictors: (Constant), 10-facts, under-10 facts
[c]Predictors: (Constant), 10-facts, under-10 facts, doubles facts
[d]Model 3: $F(3, 271) = 69.30$, $p < .01$

Table 10
Linear Regression Coefficients: 10-Facts, Under-10 Facts, and Doubles Facts as Predictors of Over-10 Facts

	Unstandardized coefficients		Standardized coefficients	t	Sig.	95% confidence interval for β	
Model	β	Std. error	β			Lower bound	Upper bound
3[a] (Constant)	–.22	.10	—	–2.13	.03	–.43	–.02
10-facts	.57	.08	.40	6.98	.00	.41	.73
Under-10 facts	.31	.08	.22	3.89	.00	.16	.47
Doubles facts	.25	.08	.17	3.34	.00	.10	.40

[a]Dependent variable: over-10 facts

Interactions Between Basic Facts, Place Value, and Word Problems

Concerns about California's singular emphasis on memorization of the first-grade basic-facts standard seem to stem in large part from a belief that mathematical proficiency requires much more than memorization. Too often, mathematics educators hear adults telling stories of feeling hopelessly lost in mathematics because they had come to see mathematics as a discipline that must be memorized, rather than understood. Some researchers have hypothesized that the ways in which students are taught the basic facts may impact the ways that students understand place value and their ability to solve problems in context. For instance, if students are taught to memorize their facts with little emphasis on conceptual understanding of the operations themselves, then those students might also be weaker in strategic competence and adaptive reasoning;[11] this weakness might be evidenced as a difficulty solving problems in context (Carpenter, Ansell, & Levi, 2001; Carpenter & Lehrer, 1999). It has also been hypothesized that students who only know how to solve problems by counting or memorization may not develop groups-of-10 understanding as effectively as students who have learned to use 10-based derived-fact strategies (Cotter, 1996, 2002).

In order to examine the possible impact of a memorization-focused standard on other important aspects of first-grade mathematical development, students were asked to solve seven questions beyond the 18 basic-fact questions (recall Figures 4 and 5). Twenty percent of the students were unable to correctly solve any of the three word problems, 54% solved one or two correctly, and 26% solved all three correctly. Although 84% of the students showed evidence of groups-of-10 under-

[11] Kilpatrick et al. (2001) described five strands that are all seen as essential for mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. According to this theory, if one or more of the five strands are neglected, then mathematical proficiency will be compromised.

standing on at least one of the four place-value problems by either counting by 10s or using place-value understanding, only 27% of the first graders demonstrated a robust conceptualization of place value on 50% or more of the groups-of-10 questions. Based on a 7-point scale, students averaged 2.5 (SD = 1.88; Mdn = 2.00).

Textbook reliance and Number Sense Proficiency. Although the state of California has banked heavily on state-adopted textbooks as a way to improve mathematics teaching and learning, the results for Number Sense Proficiency again call into question this assumption. When analyzed across all 275 students, a significant negative correlation is found between textbook reliance and Number Sense Proficiency ($r = -.13$, $p < .05$). Mirroring the results for textbook reliance correlated with Basic-Facts Retrieval and Basic-Facts Competence, this finding serves as a second indicator that the state-adopted textbooks used by the schools in this study may not be providing the type of effective instructional support that the state desires.

Instructional goals and Number Sense Proficiency. The results focused on Number Sense Proficiency in relation to instructional events beyond the textbook also seem to suggest that teachers who design their instructional programs to emphasize correct answers, hoping perhaps that repeated practice will lead to memorization of basic facts, may be reducing their students' opportunities to develop proficiency with other important aspects of the first-grade mathematics curriculum. For example, instructional events focused on getting the answer right (primarily by counting) are negatively correlated with Number Sense Proficiency ($r = -.16$, $p < .01$), whereas instructional events focused on derived-fact strategies are positively correlated with Number Sense Proficiency ($r = .19$, $p < .01$). A stepwise multiple regression also confirmed that frequency of instructional events focused on derived-fact strategies was a significant predictor of Number Sense Proficiency (Tables 11 and 12).[12]

Basic-facts solution methods and Number Sense Proficiency. From the student achievement perspective, correlational analysis also confirms that both facts retrieved from long-term memory and facts solved using derived-fact strategies correlated with higher achievement on the Number Sense Proficiency construct (retrieval: $r = .47$, $p < .01$; derived-fact strategies: $r = .34$, $p < .01$). By far the largest correlation, though, reflects the synergistic effect between memorization and derived-fact strategies: the correlation between the percentage of all addition and subtraction facts solved by retrieval or derived-fact strategies and Number Sense Proficiency was $r = .58$, $p < .01$. This finding provides support for the argument that students who learn to use derived-fact strategies in concert with memorization are more likely to develop mathematical proficiency than those students who have memorized the facts without supplementary strategies.

[12] Instructional events focused on memorization did not have significant correlations with students' use of retrieval to solve basic-fact problems or Number Sense Proficiency. A small predictive value for Number Sense Proficiency was found ($\beta = .11$, R^2 change = .01, $p < .05$).

Table 11
Linear Regression Model Summary: Pretests, Instructional Events That Are Memorization-Focused, Strategy-Focused, and Correct-Answer-Focused as Predictors for Student Number Sense Proficiency

					Change statistics				
Model	*R*	R^2	Adjusted R^2	Std. error of the estimate	R^2 change	*F* change	*df*1	*df*2	Sig. *F* change
1[a]	.45	.20	.20	24.12	.20	67.50	1	273	.000
2[b]	.47	.22	.22	23.82	.02	7.94	1	272	.005
3[c,d]	.48	.23	.22	23.69	.01	4.05	1	271	.045

[a]Predictors: (Constant), pretests
[b]Predictors: (Constant), pretests, instructional events focused on derived-fact strategies
[c]Predictors: (Constant), pretests, instructional events focused on derived-fact strategies and focused on memorization
[d]Model 3: $F(3, 271) = 27.36, p < .01$

Table 12
Linear Regression Coefficients: Pretests, Instructional Events That Were Memorization-Focused, Strategy-Focused, and Correct-Answer-Focused as Predictors for Student Number Sense Proficiency

	Unstandardized coefficients		Standardized coefficients	*t*	Sig.	95% confidence interval for β	
Model	β	Std. error	β			Lower bound	Upper bound
3[a] (Constant)	9.00	3.66		2.46	.014	1.80	16.20
Pretests	.57	.07	.42	7.77	.000	.42	.71
IEs focused on strategies	.91	.28	.18	3.21	.002	.35	1.47
IEs focused on memorization	.87	.43	.11	2.01	.045	.02	1.72

[a]Dependent variable: Number Sense Proficiency

Students who relied more heavily on counting to solve the 18 basic-fact problems, however, trended lower on Number Sense Proficiency ($r = -.48$, $p < .01$). These results seem to coincide with those related to instructional strategies. Higher levels of counting-focused instruction and higher reliance on counting to solve basic-fact problems did not seem to support Number Sense Proficiency, whereas higher levels of instruction focused on derived-fact strategies and higher use of memorization and derived-fact strategies to solve basic-fact problems were connected with enhanced Number Sense Proficiency.

Results Summary

Despite diligent instruction on the part of their teachers, barely 26% of the 275 students assessed in this study demonstrated retrieval of 50% or more of 10 addition and 8 subtraction basic facts. More than 80% of the way through the first grade, only 6.9% of the students demonstrated adequate progress on California's basic-facts standard by retrieving at least 80% of the facts. Even for students attending schools ranked academically by the state in the top 10%, achievement was far from proficient: only 10.4% retrieved at least 80% of the basic facts (M = 36.71%, SD = 27.41%, Mdn = 33.33%).

Even though neither addition nor subtraction facts retrieval were strong, subtraction was substantially weaker than addition. When accounting for both retrieval and derived-fact strategies, the difference was even more pronounced, with the median for addition more than three times that of subtraction (40.00% compared to 12.50%).

Teacher-reported textbook reliance was negatively correlated with student achievement of the standard ($r = -.14$, $p < .05$). Instead of correlating with a measure of the memorization standard, textbook reliance was positively correlated instead wth the use of counting to solve problems ($r = .14$, $p < .05$).

Teachers also reported implementing approximately 15 additional instructional activities each week, above and beyond the textbook, to help students learn their basic facts. On average, though, only 1 of these 15 events was reported by teachers to actually focus on memorization of the facts. Although the total number of instructional events did not demonstrate a statistically significant correlation with basic-fact retrieval, it did correlate positively with combined retrieval and use of derived-fact strategies ($r = .19$, $p < .01$ for addition; $r = .18$, $p < .01$ for subtraction).

In fact, teachers reported focusing an average of 8.34 instructional events each week (out of 15) on helping students learn how to use derived-fact strategies. Even with this reported focus on derived facts, though, students demonstrated limited use of these types of strategies (M = 8.36%, SD = 17.13 for addition; M = 3.23%, SD = 8.89 for subtraction). For instance, despite relatively strong doubles retrieval (74.2% for 6 + 6 and 50.5% for 7 + 7), just 55 of the 275 students used a doubles strategy on even one problem. Student use of making-10 strategies was even less frequent: Only 42 students reported any use of a 10s strategy.

Although many teachers reported emphasizing doubles memorization and doubles strategies, these did not prove to be strong predictors of retrieval of over-10 basic facts (e.g., 5 + 6 or 7 + 8). Instead, retrieval of 10-facts was a much stronger predictor (R^2 = .36 for 10-facts and R^2 = .05 for doubles facts out of a total R^2 of .43 for the model).

A number of teachers also reported using timed tests as an important part of their basic facts instructional programs. Instead of demonstrating a positive relationship, timed tests were actually identified as a slightly negative predictor of basic-fact retrieval ($\beta = -.14$, R^2 change = .02 out of an R^2 = .19 for the model, $p < .05$).

As mathematics educators, publishers, and policymakers work to refine or revise the basic-facts standard and the instructional materials designed to help students learn their facts, two instructional interactions that were found in this study should be considered. First, instruction that focused more heavily on helping students get correct answers was negatively correlated with student achievement on place value and word-problem skills ($r = -.16$, $p < .01$). This is supported by a negative correlation between students who relied more heavily on counting to solve basic-fact problems and Number Sense Proficiency ($r = -.35$ for addition and $r = -.24$ for subtraction, $p < .01$). Second, student use of derived-fact strategies in conjunction with retrieval from long-term memory demonstrated a stronger correlation with Number Sense Proficiency ($r = .58$, $p < .01$) than student use of retrieval alone ($r = .47$, $p < .01$).

DISCUSSION

This study provides evidence that a majority of California's first-grade students may be far from achieving the first stage of California's plan to accelerate mathematics learning. Five years after the adoption of accelerated standards, only 6.9% of the 275 students from nine diverse elementary schools were able to demonstrate a level of achievement on California's basic-facts memorization standard consistent with their progress through first grade (80% retrieval from long-term memory on average 83% of the way through the school year). Even with a broader interpretation of basic-facts fluency that includes problems solved using derived-fact strategies as well as retrieval from long-term memory, only 12.7% of the sampled students were at 80% or better achievement of the 10 addition and 8 subtraction fact problems.

With only 32% of the students demonstrating Basic Facts Competence on even half of the facts, it seems clear that a majority of the sampled students are not finished with this foundational standard. Without a coherent and sustained focus on addition and subtraction facts fluency during second grade, it seems likely that many of these students will not attain the automaticity that is believed to support more complex mathematical work in later grades. Yet California has no second-grade addition and subtraction basic-facts standard that would provide publishers and teachers with clear directions for a continuing focus on addition and subtraction basic facts.

The weak achievement data found in this study calls into question the feasibility of accelerating basic-facts learning. This plan is even more suspect when the data from students at the top decile schools are analyzed separately. Only 10.4% of these students were at 80% or better retrieval of the facts, and just 20.0% were at this level when derived-act strategies were included in the construct of Basic Facts Competence. With these numbers, it could be argued that the accelerated standard is unrealistic and inappropriate.

Yet international research demonstrates that first-grade students from some parts of the world seem able to achieve fluency with the addition basic facts, at the very least. Even though differences in language and educational expectations have been identified that may explain some of the basic fact learning differences between students in these countries and the United States, the findings from this study suggest that California's recommended first-grade curricula, embodied in traditional, state-approved instructional materials, are not effective in helping students to learn basic facts.

Considering the emphasis that California has placed on ensuring that students are provided with, and that teachers teach with, state-approved textbooks, it is disheartening to learn that the students whose teachers relied most heavily on state-approved textbooks achieved roughly one third as well on their basic facts (6% demonstrated 80% or better Basic Facts Competence) as students who experienced less textbook reliance (18% demonstrated 80% or better Basic Facts Competence). When teachers followed the advice of these textbooks on how best to teach their students, students were found to rely most heavily on instrumental counting, and made few gains in either their memorization or their use of derived-fact strategies. This is quite different from the data we have about students from Asian countries.

Under-10 Facts, Doubles, 10-Facts

Whereas Asian teachers have been reported to emphasize 10-based strategies, teachers in this California study seemed to focus on memorization of doubles as a way to enhance students' doubles-based strategies. Although students demonstrated more confidence with the doubles facts than any others (74% retrieval of 6 + 6 and nearly 50% retrieval of 7 + 7), they rarely seemed to use their knowledge of doubles facts to derive other related problems. Only 10.2% used a derived-fact strategy of any kind to solve 7 + 8 (15.3% retrieved from long-term memory). Even with the under-10 fact of 4 + 5, just 8.7% used a derived-fact strategy as compared with over 40% who counted to solve this under-10 problem (50.5% retrieved from long-term memory).

Another finding that aligns with practices from high-performing countries is that memorization of 10-facts served as a strong predictor of retrieval of over-10 facts such as 7 + 8 and 5 + 9; memorization of doubles did not. The prominence of the 10-facts as the single best predictor of over-10 addition fact memorization resonates with international findings. Students from high-performing countries such as China, Japan, Korea, and Taiwan have been reported to frequently use derived-fact strategies involving 10 during first grade (e.g., 7 + 8 = [5 + 2] + 8 = 5 + [2 + 8] = 5 + 10) prior to developing confident retrieval for the majority of the basic facts (Fuson & Kwon, 1992).

Although several of the teachers in this study used worksheets and timed-test materials that had students work sequentially from the +0 facts to the +10 facts, the results of this study suggest that other approaches might be more beneficial. New Zealand's Numeracy Professional Development Projects (2005, p. 41–43), for instance, recommends a basic-facts focus first on within-5s (e.g., 2 + 3) and with-5s (e.g., 5 + 4), then within-10s (e.g., 7 + 3) and doubles-to-10, then 10 numbers (e.g., 8 + 10) and doubles-to-20 (e.g., 8 + 8). This type of approach has the advantage of focusing students on part-whole relationships early on, which seems likely to support derived-fact strategies as an alternative to counting (Cotter, 1996). It also has the advantage of focusing memorization on 10-facts early on. Students in this study might have benefited from this approach; as it was, 4 + 6 was the most-often counted addition fact. Not only did 81.3% of the students not have this fact memorized, only 5.8% used a derived-fact strategy for this problem. This suggests that many students did not have the flexibility to use 5s to decompose and recompose this problem by thinking $4 + 6 = 4 + (1 + 5) = (4 + 1) + 5 = 5 + 5$.

Moving From Counting to Derived-Fact Strategies and Memorization

This study found that barely half of the students who rely on counting near the end of first grade are highly accurate (51% for addition, 46% for subtraction). For many students, counting appears to be fairly unreliable (see similar results in Geary & Bow-Thomas, 1996). From this perspective, and also considering the goal of moving beyond unitary thinking to part-whole and groups-of-10 thinking, a continuing instructional focus throughout first grade on solving basic-fact problems by counting may not be particularly beneficial for first-grade students. Certainly most students begin first grade with a need to refine their counting skills as they build a conceptual foundation for addition and subtraction. To rush through this process may jeopardize the operations and number sense that are believed to be foundational to future mathematical success. However, these data and previous research suggest that students are very likely to benefit if their teachers have derived-fact strategies and memorization as overarching goals by the end of first grade, and that they are using instructional strategies to help students transition from unitary counting to part-whole thinking.

One clear focus of instruction, used by teachers in several schools in this study, was to provide students with a physical cue to solve addition facts by counting on. Teachers explained that the instructional strategy was to have students select one of the two addends (it is hoped the larger) and say that number as they touched their heads. Then they would count on using the remaining addend, employing either fingers or mental counting. Several teachers stated that they had spent many instructional events working on this strategy, and even late in the year they were aware that not all students were confident, or accurate, counting on from the larger number. The number of students who tapped their heads as they solved problems by counting on certainly confirmed the instructional focus reported by the teachers.

From an observer's perspective, it seems possible that students and teachers saw this counting-on strategy as a desired outcome in and of itself rather than as a temporary strategy to be abandoned as soon as more efficient derived-fact strategies and memorization could be achieved. This seems to echo the lack of significant changes across time in the number of facts solved via counting that Geary and Bow-Thomas (1996) found for American first graders. In contrast, the Chinese first graders in Geary and Bow-Thomas' cross-national study made significant changes in their solution strategies, moving from a mixture of counting, decomposition, and retrieval early in first grade to primarily decomposition and retrieval later in first grade.

It appears that California's first-grade teachers and students are in a quandary. The content standard is memorization, but many students are clearly not competent solving basic-fact problems even with counting. Thus, the question remains whether or not class time and instructional resources should be devoted to improving counting strategies and accuracy first, or whether first graders can develop basic-facts fluency without completely mastering counting-on strategies. A uniform consensus has not yet been reached on this developmental question. However, Steinberg (1984) reported that at least some second graders were able to use derived-fact strategies without first having to master counting-on strategies. What does seem clear, however, is that many of the students in this study had not mastered accurate and efficient counting strategies by the end of first grade, which may have infringed on the quantity of instructional events their teachers felt comfortable focusing on memorization and derived-fact strategies.

Because California's basic-facts standard explicitly calls for memorization, it would be logical to assume that many teachers would focus their instruction on activities and tasks that are likely to aid memorization. From various conversations with teachers, it became clear that they were uncertain about how best to help their students memorize the facts. Teachers reported frequent use of worksheets, flash cards and timed tests, focusing variously on memorization, derived-fact strategies, and accuracy with counting. It seems that many of the participating teachers held a belief that repetition using counting would eventually lead to derived-fact strategies and memorization. This belief was not supported by student achievement findings. In fact, none of these three ubiquitous practice activities was found to provide a substantial contribution to memorization or fluency with the facts. Perhaps the most startling finding for teachers is that frequent use of timed tests seemed to actually work against student memorization and Basic Facts Competence.

An important question here seems to be why so many students in this study, provided with a large number of practice opportunities, still relied so heavily on counting strategies. One explanation, based on strength-of-association and distribution-of-association theories (Ashcraft, 1995; Siegler & Jenkins, 1989) could be that students' practice sessions had produced a combination of both correct and incorrect solutions such that students had developed weak associations. With weak associations, students would not have sufficient confidence for accurate retrieval. Another possibility might be that the practice opportunities did not provide students with timely feedback, so that even when students solved problems correctly, their association bonds were not strengthened because they were not certain their solutions were correct.

As teachers use various instructional strategies in hopes of developing basic-facts memorization, it is important that these strategies not inhibit the development of other foundational aspects of number sense, including operations sense and place value. Students in this study who experienced more instructional events designed to help them learn how to answer basic-fact problems correctly (generally by counting) tended to perform less well on a test of Number Sense Proficiency. The results also indicate that students who relied more on counting tended to score lower on this Number Sense Proficiency construct. Correspondingly, students who used memorization and derived-fact strategies more frequently to solve basic-fact problems, and who experienced more instructional events focused on learning how to use derived-fact strategies, tended to have higher Number Sense Proficiency scores.

In tandem, these findings seem to suggest that first graders who continue to use counting as their primary strategy throughout the year may be disadvantaged in their opportunities to develop conceptual understanding of place value and strategic competence with problems in context.

Because of the limited correspondence between the instructional events teachers identified as focusing on memorization, and the actual retrieval from long-term memory of students experiencing these events, this study is not able to provide any inferences about the effect of memory-focused instruction on Number Sense Proficiency. The finding, however, that Basic Facts Competence scores correlated more strongly with Number Sense Proficiency than did Basic Facts Retrieval scores alone, suggests that students may be better served, from a Number Sense perspective, by working simultaneously on memorization and derived-fact strategies.

These results suggest a need to identify and/or design, implement, and then study curricula that help students move beyond counting to memorization and derived-fact strategies during first grade. This may imply that less emphasis would be placed on helping students develop efficient counting strategies, and that some students might well be working on developing derived-fact strategies even before mastering counting on. As mentioned earlier, there is at least one precedent for this approach, although that research was conducted with second graders (Steinberg, 1984). The challenge in first grade, though, is to ensure that students are provided with sufficient support and experience with counting strategies to develop foundational understandings related to operations sense and number sense more generally, and then move to derived-fact strategies and memorization. Based on the finding that knowledge of 10-facts are better predictors of memorization than knowledge of doubles facts, students may benefit by engaging in a variety of activities to develop their understanding, visualization, and memorization of combinations of 10.

CONCLUSIONS

It appears that most students, even those attending high-performing schools, are far from achieving the first foundational step in accelerating mathematics learning in California. This first failure seems likely to create enormous stress on the achievement of subsequent accelerated standards that rely on number sense and computational fluency. The question that seems most pressing is whether students are unable to achieve this standard in first grade or whether the instruction they are currently receiving is misaligned with the standard.

The data from this study provide evidence that the current state-approved textbooks are encouraging reliance on counting, rather than supporting student movement away from counting and toward derived-fact strategies and memorization. Other high-frequency teacher-selected strategies such as worksheets, flash cards, and timed tests also appear to encourage continued reliance on counting. These findings are important indicators that California has not yet found the curricula and instructional strategies that could promote basic-facts fluency in both aspects of derived-fact strategies and memorization.

Although the majority of students in this study, instructed primarily via traditional textbooks and worksheets, flash cards, and timed tests, did not achieve high levels of success with a memorization standard, we have reason to believe that first-grade students can achieve addition basic facts fluency, if fluency is defined as a combination of derived-fact strategies and retrieval from long-term memory. When fluency is defined in this fashion, and when teachers are provided with curricula and instructional strategies that help students move from counting to flexible use of derived-fact strategies and memorization, students are better positioned to develop both basic-facts fluency and early number sense.

Once the research, development, and distribution of such curricula and instructional strategies have been accomplished, then we will be able to ascertain whether students can achieve both addition and subtraction facts fluency by the end of first grade. There are indications from these data, as well as international studies, that subtraction fluency may lag behind that of addition. This possibility should be easier to study once more effective basic-facts teaching is more widely implemented.

Policy and Professional Development Implications

During its 2004–2005 review of the mathematics standards and framework, California's State Board of Education elected to retain the entire body of mathematics standards without the possibility of revision or refinement. The next scheduled opportunity for reopening this discussion will be in 2011. This time frame provides a relatively short window of opportunity to engage in additional teaching and learning research, focused on other pivotal or accelerated standards across the grades, that could provide data to shape future standards revision discussions. The continuing expectation that first graders know and memorize their basic facts also leads to several challenges.

First, we are likely to have large numbers of elementary and middle school students who have not received the second- and third-grade addition and subtraction facts support that they were apt to have received pre-standards. This study's achievement data suggest that California's first-grade students, as they move forward into second grade and beyond, are far from achieving basic-facts fluency and will continue to need systematic instruction and rehearsal opportunities beyond first grade. This leads to an immediate need for teacher awareness and classroom interventions to help these students acquire the basic facts fluency they need as they study more complex forms of computation and higher levels of mathematics. Without an ongoing basic-facts instructional component in later grades, including a systematic focus on derived-fact strategies, it seems unlikely that all the students sampled in this study will be able to develop the confidence and efficiency with basic facts that is believed to be so essential to ongoing success in mathematics. This ongoing focus is most likely to occur if state-level attention is directed at the problem.

A second policy challenge in California revolves around the current state-level expectation that high reliance on state-approved textbooks will produce high student achievement on state standards. This study has provided evidence that just the opposite may be the case, at least in first-grade mathematics. The fact that high textbook reliance was actually associated with lower achievement of the standard is an indicator that traditional textbooks may not be as effective as those who have championed them have hoped. This study serves as an early warning that California's textbook adoption review process, approval procedures, and compliance requirements may need serious study. In addition, the same expectations for research-based evidence of student achievement should be applied to traditional textbooks as those that are being applied to newer reform-based instructional materials.

The issue of textbook insufficiency is also immediately important to first-grade students in the next several years until new, more effective materials can be identified, approved, and purchased for classrooms. The results of this study indicate that teachers may have a limited understanding of the types of instructional events that are likely to support memorization and part-whole thinking in support of derived-fact strategies. Although the instructional events they identified as focusing on memorization and derived-fact strategies may have been slightly effective in improving the use of derived-fact strategies and to a lesser degree in reducing the use of counting, they do not appear to have been effective in helping students increase their use of retrieval from long-term memory. The finding that in-class repeated practice (as self-reported by teachers) did not positively correlate with retrieval from long-term memory raises questions about the instructional practices that assist students in developing bonds of association (Ashcraft, 1990, 1995; Siegler & Jenkins, 1989) between basic-fact combinations and their sums and differences. This points to the need for both research and professional development for first-grade teachers to help them understand best practices in relation to teaching basic facts. State-level support for both research and professional development can ensure that all first-grade students have improved opportunities to develop foundational number sense and fluency.

This study's findings may prove useful to teachers, professional development trainers, and textbook publishers as they consider ways to improve basic-facts learning. At least for the present, it makes sense for teachers and professional developers to develop a healthy skepticism for the pacing and instructional strategies recommended in at least some state-approved textbooks. Not only did students in this study not make strong progress toward the basic-facts standard, even with quite high textbook reliance and many extra instructional events, textbook reliance was to some extent negatively correlated with basic-facts memorization and the use of derived-fact strategies.

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